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2002 MALAWI SCHOOL CERTIFICATE OF EDUCATION EXAMINATION

MATHEMATICS SOLUTIONS

1. Simplify $\frac{(3\tfrac{1}{2} \times 1\tfrac{1}{2}) - 3}{9}$

Solution

$\frac{(3\tfrac{1}{2} \times 1\tfrac{1}{2}) - 3}{9}$

$= \frac{\left(\frac{7}{2}\times\frac{3}{2}\right)-3}{9}$

$= \frac{\left(\frac{21}{4}-3\right)}{9}$

$= \frac{5\ \ -3}{9}$

$= 2\tfrac{1}{4} \div 9$

$= \frac{9}{4} \div \frac{9}{1}$

$= \frac{9}{4} \times \frac{1}{9}$

$= \frac{1}{4}$ **Answer**

2. Figure 1 is a pie chart representing sales of three commodities; tobacco, tea and coffee.

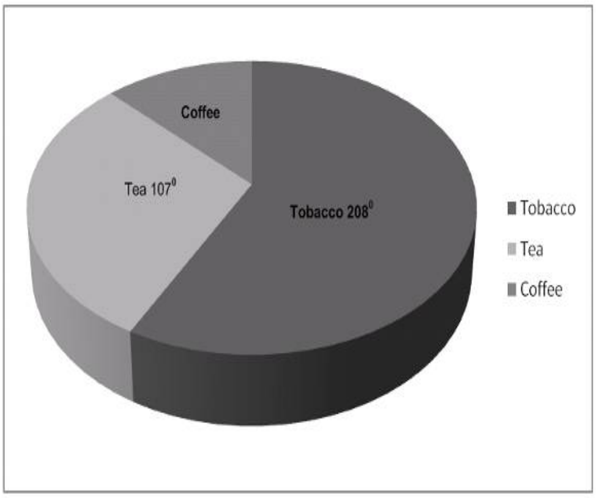


Figure 1

Express coffee sales as a percentage of the total sales.

Working

Angle sector for coffee= $360^0-(208^0+107^0)$

$=360^0-315^0$

$=45^0$

∴ Percentage= $\frac{45^0}{360^0} \times 100^{25}$
8
2

=12½ % Answer

3. P is a point on the graph whose equation is $y=x^2-6x$. If the x-coordinate of P is 2, calculate its y coordinate.

Solution

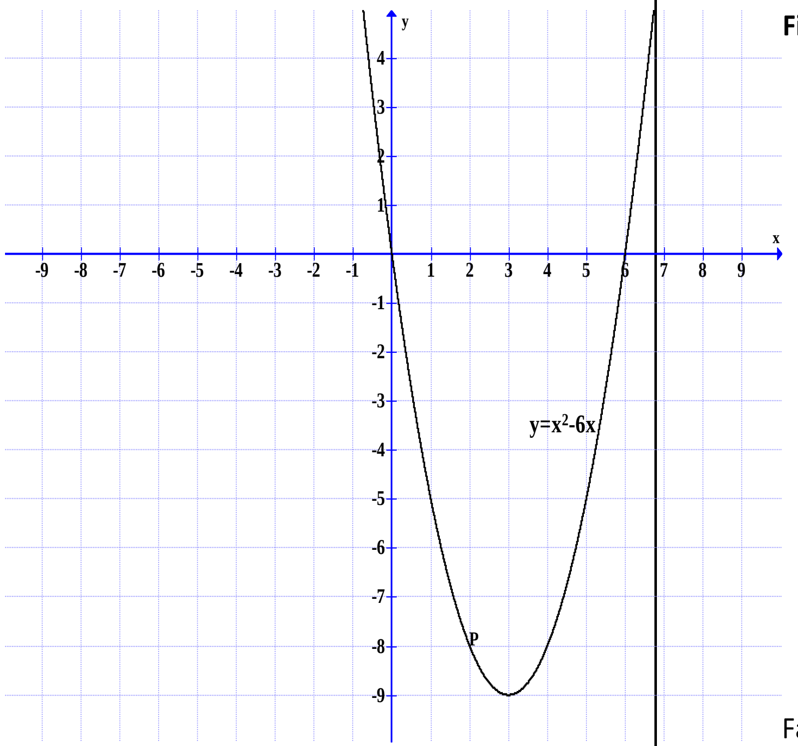


Figure 2

When x is 2 at P, $y=(2)^2-6(2)$

$y=4-12$

$y=-8$

∴ y coordinate of P is= 8 Answer

4. In a cyclic quadrilateral ABCD twice angle BAD=three times angle DCB. Calculate angle BAD.

Solution

Sketch

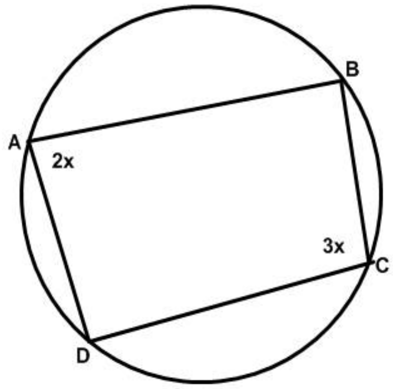


Figure 3

Let angle BAD be 2x and angle DCB be 3x.

$2x+3x=180^0$(opposite angles of cyclic quadrilateral are supplementary)

Thus $5x=180^0$

$X=180^0 \div 5$

$=36^0$

∴ Angle BAD$=36^0 \times 2$

$=72^0$ Answer.

5. Factorise completely $1-16(1-y)^2$

Solution

Factorizing $1-16(1-y)^2$

$=(1)^2-(4)^2(1-y)^2$

$=\{1+4(1-y)\}\{1-4(1-y)\}$

$=\{(1+4-4y)(1-4+4y)\}$

=(5-4y)(-3+4y) Answer

6. In figure 4, DB is perpendicular to the line ABC, AE=25cm, BC=15 cm, angle EAB=30^0, and angle BCD=45^0. Calculate the length of DE.

Solution

$=\frac{BE}{25cm}=\text{Sin } 30^0$

$=BE= \text{Sin } 30^0$

$=0.5\times25$

$=12.5cm$

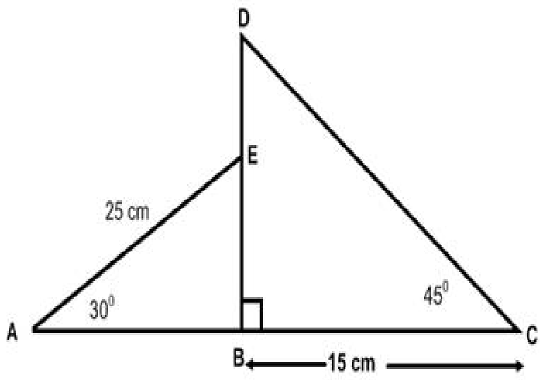


Figure 4

$=\frac{BD}{15cm}$=Tan 45^0

BD=Tan 45^0×15 cm

=1 ×15 cm

=15 cm

∴ DE= (15-12.5) cm

7. Given that $\frac{a^7}{a^{-3}\times a^2} = a^y$, Find the value of y.

Solution

$\frac{a^7}{a^{-3}\times a^2} = a^y$

$a^{7-(-3)-2}$= a^y(law of indices)

a^{7+3-3}=a^y

Thus a^{10-2}=a^y

a^8=a^y

Equate powers

∴y=8 Answer

8. In Figure 5, D is the midpoint of the minor arc BDC, angle ABC=40^0 and angle ACB=60^0. Calculate angle DAC.

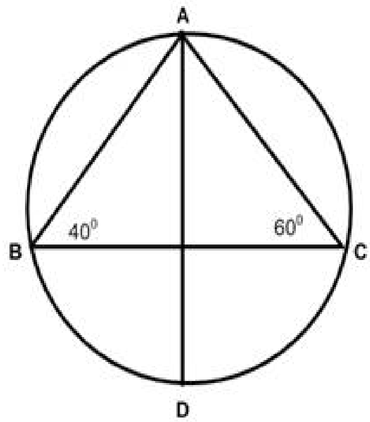


Figure 5

Solution

Construction: Join BD, CD.

D is midpoint (given)

Thus bisects chord BC at DI.

BK=CK.

ABC must intersect AD at right angles.

Angle CDA=CBA=40^0(angles in same segments)

Also angle ACB=angle ADB=60^0(angles in same segment)

∴ In Δ BDK, angle DBK=180^0-(60^0+90^0)

=180^0-150^0

=30^0

But angle DBK and DAC are in the same segment.

∴ Angle DAC=30^0 Answer.

9. Make x the subject of the formula 4y=a^x

Working

4y= a^x

(Change to logarithmic equation)

Log_a 4y = x Answer

10. In figure 6, O is the centre of the centre of the circle, TA is a tangent, BC is parallel to TA and angle BTC=37^0. Calculate the value of the angle marked y.

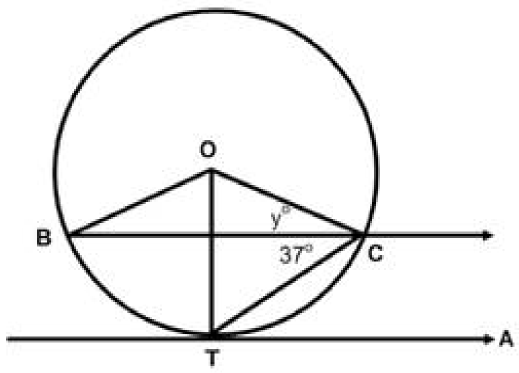


Figure 6

Solution

Angle OTA is right-angled (OT is perpendicular to Tangent TA)

OC=OT=OB (Radii)

Angle BCT=Angle CTA (alternate angles)

Therefore angle CTA=37^0

Angle OTC= 90^0-37^0(OT perpendicular to Tangent TA)

=53^0

Angle OTC = Angle TCO (base angles of isosceles triangle)

Angle OTC=y^o+37^0

y^0+37^0=53^0

y^0=53^0-37^0

y^0=16^0 Answer

11. Solve for x $Log_x 125^{-1}$=-3

Working

$Log_x 125^{-1}$=-3

(Change to exponential equation)

125^{-1}=x^{-3}

$5(3)^{-1}=x^{-3}$
$5^{-3}=x^{-3}$
Equate bases
∴ x=5 Answer

12. Given that 2x, x, x+3, ……are terms in an Arithmetic Progression. Calculate the value of x.
Solution
nth term
a+(n-1)d where
a=First term
d= common difference
n= No of terms
2th term= a+(n-1)d
=2x+d(2-1)=x
=2x+d(1)=x
d=x-2x
=-x
3^{rd} term= 2x+-x(3-1)= x+3
2x +-x(2)=x+3
2x-2x=x+3
-x=3
x= -3 Answer
Proof
(2 ×-3, -3, -3+3…)
(-6, -3, 0)

13. **Simplify $\sqrt[3]{8a}+2\sqrt[3]{a}-\sqrt[3]{27a}$ Giving your answer in the simplest form**
Working
$\sqrt[3]{8a}+2\sqrt[3]{a}-\sqrt[3]{27a}$
$\sqrt[3]{8}\times\sqrt{a}+2\sqrt[3]{a}-\sqrt[3]{27}\times\sqrt[3]{a}$
$=2\sqrt[3]{a}+2\sqrt[3]{a}-3\sqrt[3]{a}$
$=4\sqrt[3]{a}-3\sqrt[3]{a}$
$=\sqrt[3]{a}$ **Answer**

14. In Figure 5, AB is parallel to CD, EF and GH. The parallel lines AB, EF and GH intersect QR such that QX=XY=YR. SU= 9 cm, DU=8cm and TV=5 cm. Prove that angle TUV=90^0.
Solution
AB//CD//BP//EF//GH (Given)
QX=XY=YR
$∴ \frac{3}{3}TU=\frac{9cm}{3}$
TU=3 cm
And $\frac{2UV}{2}=\frac{3cm}{2}$
UV=4 cm
∴ $(TV)^2=(UV)^2+(TU)^2$ (Pythagoras Theorem)
$(5\text{ cm})^2=(4\text{ cm})^2+(3\text{ cm})^2$
$25cm^2=25cm^2$
∴ Angle TUV=90^0 Answer

15. The number of people (N) who suffer from Malaria in a month is inversely proportional to the amount of insecticides (M) applied that month. When 5 litres of insecticide are applied, only 1 person suffers from Malaria. Find the equation connecting N and M.
Solution
$N\propto\frac{1}{M}$

$N=\frac{K}{M}$ (K constant)
$I=\frac{K}{5}$ (Multiply both sides by 5)
∴ k= 5
∴ Equation is $N=\frac{5}{M}$ Answer

16. P is a set of points (x,y) which satisfies the three inequalities: x≥0, x+y≤4, y≥x=1
Solution
Lines: (1) x=0
(2) x+y=4
Coordinates $(x=0,4\ \ y=4,0\ )$
(3) y=x+1
Coordinates $(x=0,4\quad y=4,0)$

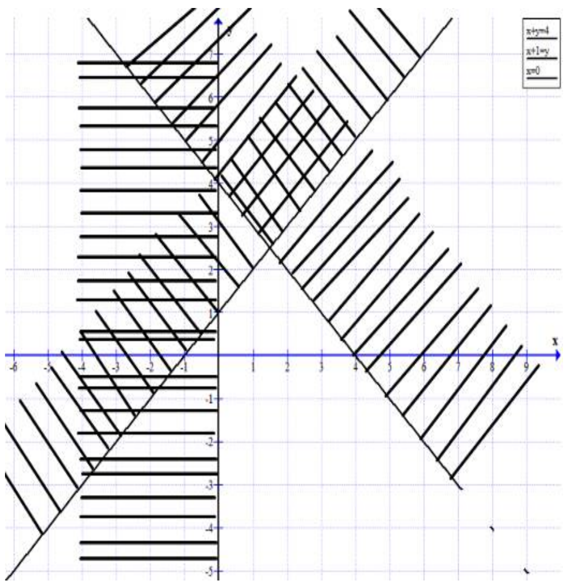


Figure 7

17. A straight line passes through points A(1,-1) and B(7,-9). Calculate the distance between A and B.
Solution
A(1,-1)
B(7,-9)
Distance between A and B
$=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(7-1)^2+(-9-(-1)^2}$

$=\sqrt{6^2+(-8)^2}$

$=\sqrt{36+64}$

$=\sqrt{100}$

=10 Answer

18. The volume of a cone is 462 cm3. If its height is 9 cm. Calculate it radius.

(Taking $\pi=\frac{22}{7}$ and volume of a cone $=\frac{1}{3}\pi r^2 h$

Solution

462 cm^3$=\frac{1}{3}\pi r^2 h$

462 cm^3$=\frac{1}{3}\times\frac{22}{7}\times r^2\times 9\text{ cm}$

~~21~~ 7 1

$=\frac{462\times7\times3}{22\times9\ 3}=r^2$

r^2= 7 × 7

r=7 cm Answer

19. In figure 7 triangle ABC is similar to triangle BAD. If the area of triangle ABC =72 cm^2, area of triangle BAD=200 cm^2, and BC=6 cm, calculate the length of AD.

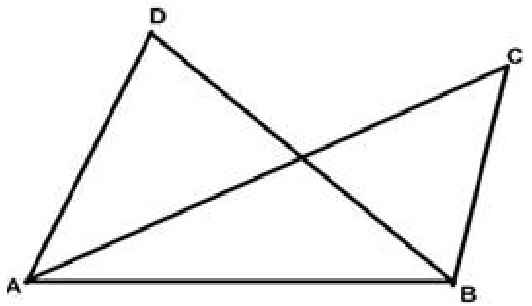


Figure 8

Solution

Triangles ABC and BAD are ≡ (Given)

$\frac{AB}{BA}=\frac{BC}{AD}=\frac{CA}{DB}$

$\frac{72\text{cm}^2}{200\text{cm}^2}=\frac{(6\text{cm})^2}{(AD)^2}$ (Area factor is in scale factor)

$\frac{72\text{cm}^2}{200\text{cm}^2}=\frac{6\text{cm}\times6\text{ cm}}{(AD)^2}$

$72\text{cm}^2 AD^2=36\times200$

100

$AD^2=\frac{36\times200}{72}$

~~2~~

~~1~~

AD^2=100

AD= $\sqrt{100}$

=10 cm Answer

20. In figure 9, triangle ABC is isosceles in which AB=AC and angle BAC=140^0. AB and AC are produced to D and E respectively. The bisectors of angle CBD and angle BCE meet at O.

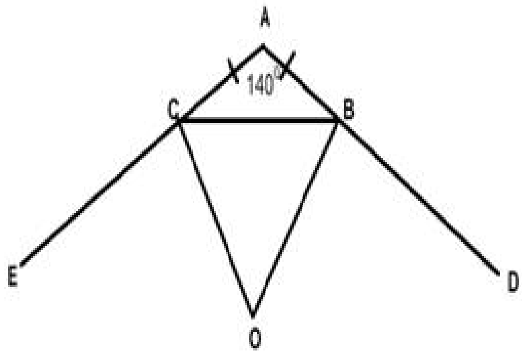


Figure 9

Solution

Triangle ABC isosceles(given)

Base angles are (180^0-140^0)÷2

=40÷2

=20^0

Lines OC and OB bisect angles ECB and DBC respectively

=(180^0-20^0) ÷2

=160^0÷2

=80^0

In Triangle BCO, angle BCO angle CBO=80^0

∴ Angle BOC= 180^0-160^0

=20^0 Answer

21. When a polynomial **x^3+kx^2+x-k** is divided by **x-k,** the remainder is **2**. Calculate the value of **k**.

Solution

Let x-k=0

X=k

$(k)^3+k(k)^2+k-k$

$k^3+k^3+0=2$

$2k^3=2$

$k^3=1$

$k=\sqrt[3]{1}$

k=1 Answer

2003 MALAWI SCHOOL CERTIFICATE OF EDUCATION EXAMINATION

MATHEMATICS SOLUTIONS

1. Factorise completely $x^2+3x+4(x-3)$

   Solution

   $x^2+3x+4(x-3)$

   $x^2+3x+4x+12$

   $x^2+7x+12$

   Multiply 12 by x^2

   $=12x^2$(Factors are +3x +4x)

   =(x+3)(x+4) Answer

2. Given that f(x)=x3-x, calculate f(-2)

   Solution

   $f(-2)=(-2)^3-(-2)$

   f(-2)=-8-(-2)

   f(-2)=-6 Answer

3. Express $\frac{3}{\sqrt{2}}$ as a fraction with a rational denominator.

   Solution

   $=\frac{3}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

   $=\frac{3\sqrt{2}}{2}$ **Answer**

4. Given that

   $a=\begin{pmatrix}3 & 0\\ -4 & 4\end{pmatrix}$ and $b=\begin{pmatrix}2 & -1\\ -1 & 0\end{pmatrix}$, calculate ab.

   Solution

   $ab=\begin{pmatrix}3 & 0\\ -4 & 4\end{pmatrix}\begin{pmatrix}2 & -1\\ -1 & 0\end{pmatrix}$

   $=$

   $\begin{pmatrix}3\times\ 2+0\times -1 & 3\times -1+0\times 0\\ -4\ \times 2+4\times -1 & -4\times -1+4\times 0\end{pmatrix}$

   $=\begin{pmatrix}\mathbf{6} & \mathbf{-3}\\ \mathbf{-12} & \mathbf{4}\end{pmatrix}$ **Answer**

5. The universal set (∈)= {10,20,30,40,50,60,70}, A={10,30,60} and B={20,40,50}, evaluate A'∩B.

   Solution

   A={10,30,60}

   B={20,40,50}

   A'={20,40,50,70}

   ∴A'∩ B= {20, 40, 50} Answer

6. A point T has the coordinates $\begin{pmatrix}3\\2\end{pmatrix}$. The matrix which transforms T into T'is $\begin{pmatrix}2 & 0\\ 1 & 1\end{pmatrix}$. Calculate the coordinates of T'.

   Solution

   IV= (TV)(OV)

   $=\begin{pmatrix}3\\2\end{pmatrix}\begin{pmatrix}2 & 0\\ 1 & 1\end{pmatrix}$

   $=\begin{pmatrix}3\times 2+3\times 1 & 3\times 0+3\times 1\\ 2\times 2+2\times 1 & 2\times 0+2\times 1\end{pmatrix}$

   $=\begin{pmatrix}9 & 3\\ 6 & 2\end{pmatrix}$ Answer

7. Calculate vector $\overrightarrow{AB}$ if vectors A=$\begin{pmatrix}3\\5\end{pmatrix}$ and B=$\begin{pmatrix}-4\\4\end{pmatrix}$.

   Solution

   $\overrightarrow{AB}=\begin{pmatrix}-4\\4\end{pmatrix}-\begin{pmatrix}3\\5\end{pmatrix}$

   ∴Vector $\overrightarrow{AB}$= $\begin{pmatrix}-7\\-1\end{pmatrix}$ Answer

8. Given $\log_a 2$=0.6110 and $\log_a 3$=0.7039, calculate $\log_a 6$.

   Solution

   $\log_a 6=\log_a 2\times\log_a 3$

   Rule of logarithm ($\log_a mn=\log_a m+\log_a n$)

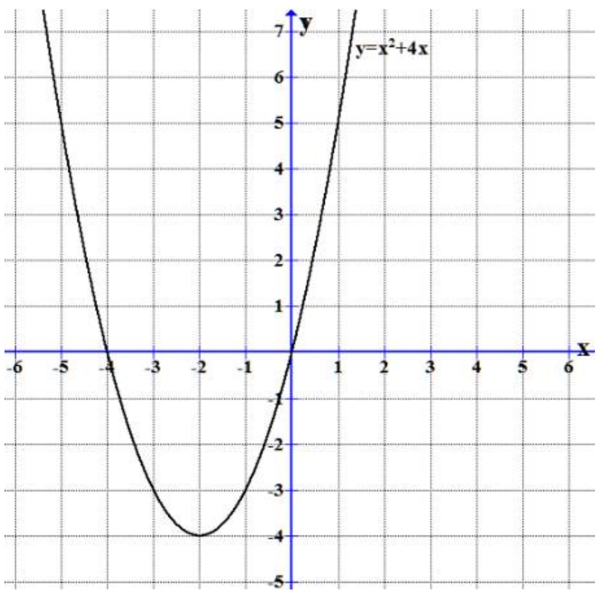
   $\log_a 6=\log_a 2+\log_a 3$

   $\log_a 6$ **=0.6110+0.7039**

   ∴$\log_a 6$ =1.3149 Answer

9. Calculate the coordinates of the turning point on the curve $y=x^2+4x$

   Figure 1

   Solution

   Let 'y' be 0

   $0=x^2+4x$

$-4x = -4x$

$\frac{-4\cancel{x}}{\cancel{x}} = \frac{\cancel{x}^2}{\cancel{x}}$

∴x=-4 Answer

y=-4(-4+4)

y=-4(0)

∴y=-4(0)

∴y=0 Answer

10. Express $\frac{1}{x^2-x-2}$ - $\frac{1}{x+1}$ as a single fraction

Solution

$\frac{1}{x^2-x-2} - \frac{1}{x+1}$

$= \frac{1-(x-2)}{(x+1)(x-2)}$

$= \frac{1-(x+2)}{(x+1)(x-2)}$

$= \frac{1+2-x}{(x+1)(x-2)}$

$= \frac{3-x}{(x+1)(x-2)}$ **Answer**

11. Make *m* the subject of the formula $y = \frac{m}{1+m}$

Solution

$y = \frac{m}{1+m}$

$(1+m)\, y = \frac{m\cancel{(1+m)}}{\cancel{(1+m)}}$

y+my=m

y+my-my=m-my

y=m-my

$\frac{y}{1-y} = \frac{m\cancel{(1-y)}}{\cancel{(1-y)}}$

∴m= $\frac{y}{(1-y)}$ **Answer**

12. The line joining the points A (3, q) and B (5-q,8) has a gradient of $\frac{1}{2}$. Calculate the value of q.

Solution

A(3,q) and B(5-q,8)

Gradient= $\frac{1}{2}$

Gradient= $\frac{y_2-y_1}{x_2-x_1}$

m= $\frac{y_2-y_1}{x_2-x_1}$

$\frac{1}{2} = \frac{8-q}{(5-q)-3}$

$\frac{1}{2} = \frac{8-q}{5-q-3}$

$\frac{1}{2} = \frac{8-q}{2-q}$

1(2-q)=2(8-q)

2-q=16-2q

2-q+q=16-2q+q

2=16-q

q=16-2

q=14 Answer

13. Given that **x** varies jointly as **y** and inversely as the square of **z**, calculate the missing value in **Table 1.**

Table 1

x	y	z
3	1	2
1	3	

Solution

$x \propto \frac{y}{z^2}$

$x = \propto \frac{ay}{z^2}$ where a is constant

$x = \frac{ay}{z^2}$

$3 = \frac{a(1)}{2^2}$

$4 \times 3 = \frac{a \times 4}{2^2}$

a=12

$x = \frac{12(3)}{z^2}$

$1 = \frac{36}{z^2}$

$z^2 = 36$

$\sqrt{z^2} = \sqrt{36}$

∴z=6 Answer

14. A box contains 5 red balls, 8 white balls and 7 black balls. If one ball is selected at random, calculate the probability that is white or black.

Solution

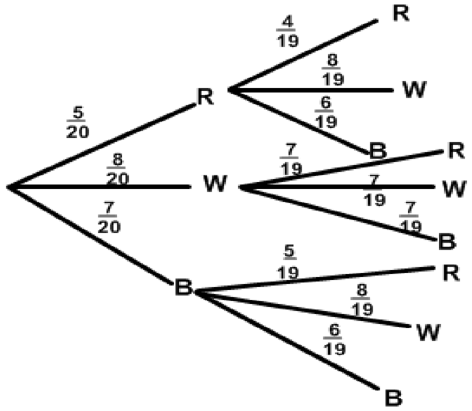


Figure 2

Probability that it is white or black

(WB)+ (WB)+ (WB)

$\left(\frac{8}{20}\times\frac{6}{9}\right)+\left(\frac{8}{20}\times\frac{7}{19}\right)+\left(\frac{8}{20}\times\frac{6}{19}\right)$

$\frac{48}{380}+\frac{56}{380}+\frac{48}{380}$

$\frac{152}{380}=\frac{4}{10}=\frac{2}{5}$

∴ Probability that it is White or Black

$\frac{2}{5}$ **Answer**

15. Table shows the distribution of the number of employees in 43 factories in a town. Draw a histogram.

Number of Employees	0-39	40-59	60-79	80-99
Number of Factories	5	15	13	10

Table 4

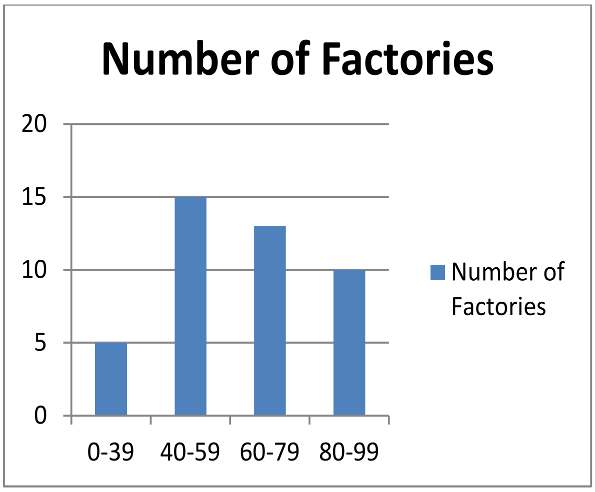


Figure 3

16. A circle centre O has a tangent PA at a point A. AT is a chord such that angle TAP is acute. If angle TAP is 700. Calculate the value of angle OTA

Solution

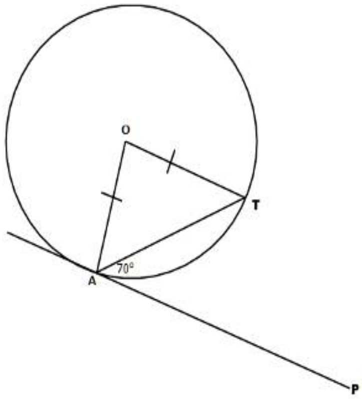


Figure 4

Solution

Angle TAO+Angle TAP=70^0(Tangent to a circle is perpendicular)

Angle TAO=90^0-70^0

=20^0

Angle OTA=Angle TAO(base angles of triangle OAT)

∴Angle OTA=20^0 Answer

17. Three terms of GP=x+1, x^2-1 and (x^2-1)(2x-4) Calculate the value of x.

Solution

GP=ar^{n-1}

a=x+1

n=3

$r=\frac{x^2}{x+1}$

$r=\frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}}$

r= (x-1)

GP=(x+1)$(x-1)^{3-1}$

=$(x^2-1)^2$

=$(x^2-1)(x^2-1)$

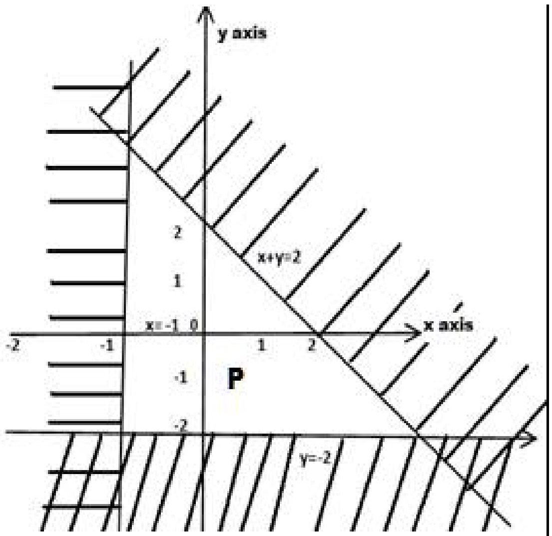
Equation

$\sqrt{x^2}=\sqrt{1}$

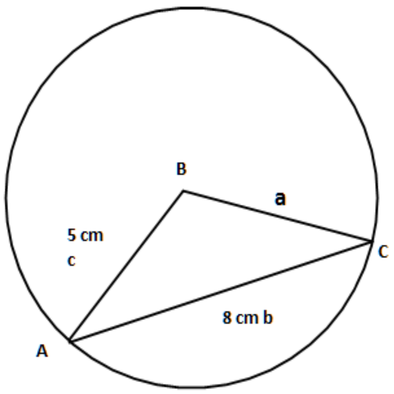
∴x=1 or -1 Answer.

18. P is a set of points(x,y) which satisfies the three inequalities:

x>-1, y>-2, x+y<2. Using a scale of 2cm to represent 1 unit on the x-axis and y-axis draw the region P.



19. A chord of a circle of radius 5 cm is 8cm long. Sketch the diagram and calculate the angle subtended by the chord at the centre of the circle



Solution

AB=AC(equal radii)

∴AC=5 cm

Cos B=$\frac{c^2+a^2+b^2}{2ca}$

=$\frac{5^2+5^2-8^2}{2(5\times5)}$

= $\frac{-14}{50}$

= -0.28

∴B=Cos -0.28

B=106.26^0 Answer

20. The figure below shows a rectangular box with an open top. The box measures 6 cm long, 2x cm wide and x cm high. Given that the total outer surface area of is 108cm^2. Form an equation in x and show that it simplifies to $x^2+6x-27=0$

Solution

Area of a rectangle = Length × breadth

Substitution

$108=4x^2+24x$

$4x^2+24x-108=0$

$x^2+6x-27=0$ Answer

21. Find the remainder when $2x^3-13x^2-8x+12$ is divided by 2x-1

Solution

x^2-6x-7

2x-1 | $2x^3$-13x^2-8x+12

(-) $2x^3$-x^2

-12x^2-8x

(-)-12x^2+6x

-14x+12

(-) -14x+ 7

5

∴When $2x^3-13x^2-8x+12$ is divided by 2x-1 the remainder is 5. Answer

2004 MALAWI SCHOOL CERTIFICATE OF EDUCATION EXAMINATION

MATHEMATICS SOLUTIONS

1. Factorise completely $6+x-2x^2$.

Solution

$6+x-2x^2$

Multiply $-12x^2$ by 6 (factors +4x-3x)

$=6+4x-3x-2x^2$

=2(3+2x)-x(3+2x)

=(3+2x)(2-x) Answer

2. A straight line passes through the point (1,6) and cuts the y-axis at 4, calculate its gradient.

Solution

Gradient/G= $\frac{\text{Change in y}}{\text{Change in x}}$

$\frac{y_2-y_1}{x_2-x_1}$

By substituting= $\frac{4-6}{0-1}$

=$\frac{-2}{-1}$

= 2

∴ Gradient= 2 Answer

3. Given that g(x) =$\frac{3x}{x+1}$, calculate the value of x when g(x)=2

Solution

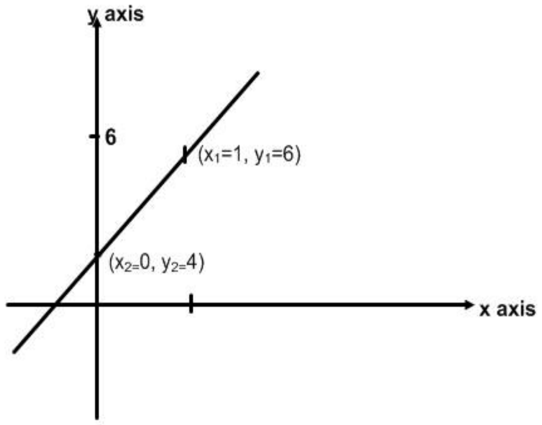


Figure 1

G(x)= $\frac{3x}{x+1}$ when g(x) is 2, it follows that

$\frac{2}{1}$ =$\frac{3x}{x+1}$ (Cross multiply)

1×(3x)= 2×(x+1)

3x=2x+2

3x-2x=2

∴ x= 2 Answer

4. A quality b varies jointly with r and t, and b=108 when r=3 and t=6. Find an equation which expresses b in terms of r and t.

Solution

b∝rt

b=krt where k is a constant

108= k×3×6

$\frac{108}{18}$ =$\frac{18k}{18}$

6=k

∴k=6

∴ An equation is b= 6 r t Answer

5. In figure 1, ABC is a triangle in which angle BAC=90^0, angle ABC=27^0 and AB=5 cm. Calculate the length of AC.

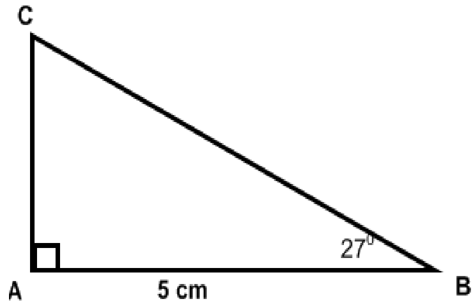


Figure 2

Solution

Triangle ABC is right angled at A.

AC is opposite side to angle ABC(27^0)

AB is adjacent to angle ABC (27^0)

∴$\frac{\text{Opposite}}{\text{Adjacent}}$=Tan 27^0

$\frac{AC}{AB}$ =Tan 27^0

$\frac{A\,C}{5cm}$ = Tan 27^0

AC = Tan 27^0×5cm

= 0.5095× 5 cm

=2.5 cm Answer.

6. Make y the subject of the formula $\frac{y+d}{c} = \frac{3d}{y-d}$

Solution

$\frac{y+d}{c} = \frac{3d}{y-d}$ (Cross multiply)

(y-d) (y+d)= c (3d)

y^2+yd-yd-d^2=3cd

y^2-d^2=3cd

y^2=3cd+d2

y^2=d (3c+d) (Take $\sqrt{\ }$ on both sides)

y=$\sqrt{d(3c+d)}$ Answer

7. Given that $\begin{pmatrix}3c & c\\5 & 1\end{pmatrix}\begin{pmatrix}4\\2\end{pmatrix}$=$\begin{pmatrix}28\\22\end{pmatrix}$, Find c.

Solution

$\begin{pmatrix}3c & c\\5 & 1\end{pmatrix}\begin{pmatrix}4\\2\end{pmatrix}$=$\begin{pmatrix}28\\22\end{pmatrix}$

$\begin{pmatrix}3c \times 4 + c \times 2\\5 \times 4 + 1 \times 2\end{pmatrix}$ =$\begin{pmatrix}28\\22\end{pmatrix}$

$\begin{pmatrix}12c + 2c\\20 + 2\end{pmatrix}$ =$\begin{pmatrix}28\\22\end{pmatrix}$

$\begin{pmatrix}14c\\22\end{pmatrix}$ =$\begin{pmatrix}28\\22\end{pmatrix}$ (Equate corresponding components)

14c= 28

C= 2 Answer (by dividing both sides by 14)

8. The sum of the first two terms of a geometric progression is 4. If the first term is 3, find the common ratio.

Solution

$S_n = \frac{a(r^n-1)}{r-1}$ (a= First term, n= no of terms, r= common ratio)

$4 = \frac{3(r^2-1)}{r-1}$

$\frac{4}{3} = \frac{r^2-1}{r-1}$

$\frac{4}{3} = \frac{(r+1)\cancel{(r-1)}}{\cancel{(r-1)}}$

$\frac{4}{3}$ =r+1

$1\frac{1}{3}$= r+1 (collect like terms)

$1\frac{1}{3}$ -1=r

∴ r= $\frac{1}{3}$

∴ Common ratio is $\frac{1}{3}$ Answer.

9. The areas of two similar triangles ABC and HKL are $100cm^2$ and $256cm^2$ respectively. If the length of AB is 5 cm, calculate the length of HK.

Solution

Triangles ABC and HKL are similar.

∴$\frac{AB}{HK} = \frac{BC}{KL} = \frac{CA}{LK}$

$\frac{5cm}{HK} = \frac{100cm^2}{256cm^2}$

But areas of similar triangles are in ratio of squares of corresponding sides.

∴$\frac{(5\ cm)^2}{(HK)^2} = \frac{100cm^2}{256\ cm^2}$

$\frac{25cm^2}{HK^2} = \frac{100cm^2}{256cm^2}$ (cross multiply)

$100cm^2 \times HK^2 = 256cm^2 \times 25cm^2$

Make HK^2 subject of the formula.

$(HK)^2 = \frac{256cm^2 \times \cancel{25cm^2}}{\cancel{100cm^2}_{4}}$

$(HK)^2 = \frac{256cm^2}{4}$

$(HK)^2 = 64cm^2$ (Take square roots on both sides)

$\sqrt{(HK)^2} = \sqrt{64cm^2}$

∴ HK= 8 cm Answer

10. Figure 3, shows an unshaded region bounded by three inequalities.
Write down the three inequalities.

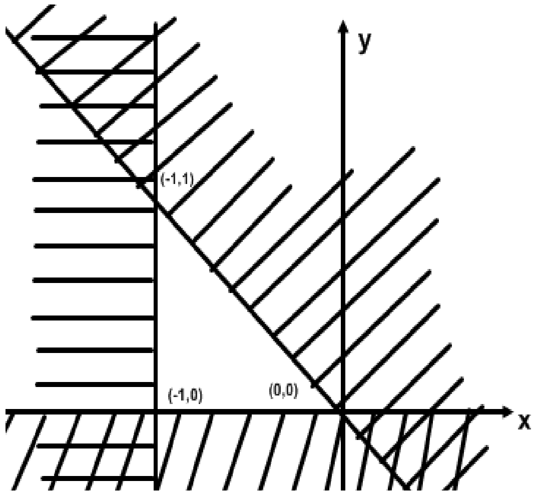


Figure 3

Solution

(a) x≥-1 (This is a vertical line parallel to y-axis. Line is continous and shaded at its back)

(b) y≥0 (line is continous and shaded below y=0 along x-axis)

(c) For the line passing through (-1,1) and (0,0)

(d) Gradient is= $\frac{1-0}{-1-0}$

$= \frac{1}{-1}$

=-1

Inequality is $y-y_1 \leq m(x-x1)$

y-1≤-1(x-(-1)

y-1≤-1(x+1)

y-1≤-x-1

y≤-x-1+1

y≤-x

Because the line passing through (-1,1) and (0,0) has been shaded above it, thus the region of interest is below this line. At the same time, the lines are all solid or continous, therefore values are all part of solutions.

11. A chord is 6 cm from the centre of a circle. If the chord is 16 cm long, calculate the radius of the circle.

Solution

Sketch

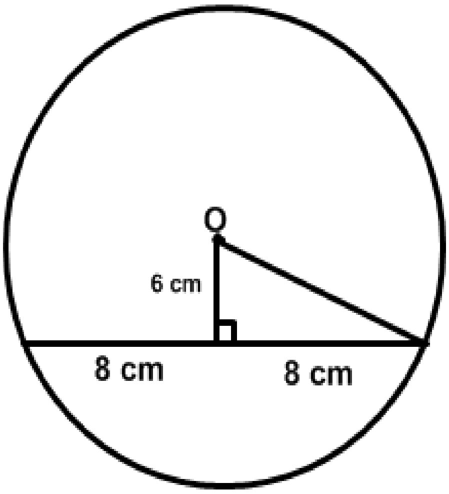


Figure 4

A line passing through centre bisects chord.

$(\text{radius})^2=(62+82)\ cm^2$

$=36+64\ cm^2$. (Take $\sqrt{\ }$ on both sides)

$r^2=100cm^2$

∴ Radius= 10 cm Answer.

12. Simplify $\frac{\sqrt{2}}{\sqrt{2}-1}$

Solution

$\frac{\sqrt{2}}{\sqrt{2}-1}$ (Multiply both numerator and denominator by conjugate of denominator) $\sqrt{2+1})$

$=\frac{\sqrt{2}\times\sqrt{2}+1}{(\sqrt{2}-1)\times(\sqrt{2}+1)}$

$=\frac{2+\sqrt{2}}{2+\sqrt{2}-\sqrt{2}-1}$

$=\frac{2+\sqrt{2}}{2-1}$

$=2+\sqrt{2}$ Answer

13. Given that $\underset{AB}{\rightarrow}\begin{pmatrix}3\\5\end{pmatrix}$and $\underset{BC}{\rightarrow}\begin{pmatrix}\mathbf{9}\\\mathbf{15}\end{pmatrix}$ are parallel. They have a common point and they must be collinear.

Solution

$\underset{AB}{\rightarrow}\times 3=\underset{BC}{\rightarrow}$

i.e. $\begin{pmatrix}3\\5\end{pmatrix}\times 3=\begin{pmatrix}9\\15\end{pmatrix}$

Also $\underset{AB}{\rightarrow}\times1=\underset{AB}{\rightarrow}$

i.e$\begin{pmatrix}3\\5\end{pmatrix}\times1=\begin{pmatrix}3\\5\end{pmatrix}$

∴$\underset{AB}{\rightarrow}$ and $\underset{BC}{\rightarrow}$ are parallel. They have a common point and they must be collinear.

14. Express $\frac{m-2}{m-3}+\frac{m+3}{m+2}$ as a single fraction in its lowest term.

Solution

$\frac{m-2}{m-3}+\frac{m+3}{m+2}$ (Common denominator is (m-3)(m+2))

$=\frac{(m-2)(m+2)+(m+3)(m-3)}{(m-3)(m+2)}$

$=\frac{m^2+2m-2m-4+m^2-3m+3m-9}{(m-3)(m+2)}$

$=\frac{2m^2-4-9}{(m-3)(m+2)}$

$=\frac{\mathbf{2m^2-13}}{\mathbf{(m-3)(m+2)}}$ **Answer**

15. In figure 5, TX and TY are tangents to the circle XHY at X and Y.

If angle XHY=70^0, calculate angle XTY.

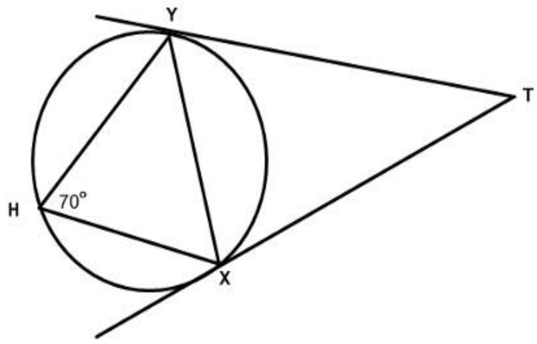


Figure 5

Solution

Angle XHY=Angle TXY=70^0 (angles in alternate segments)

But TY=TX (Tangents from an external point to the circle are equal)

∴ Triangle TXY is isosceles.

∴ Angle TXY=Angle TYX (base angles of isosceles triangle)

∴Angle XTY= $(180^0-(70^0+70^0)$ (angle sum in Triangle)

$=180^0-140^0$

$=40^0$ Answer.

16. Use the remainder theorem to prove that (x-y) is a factor of a polynomial $x^2(y-2)+y^2(2-x)$

Solution

Let (x-y)=0

x=y

Substitute x by y in the polynomial $x^2(y-2)+y^2(2-x)$

$=y^2(y-2) +y^2 (2-y)$

Expanding yields

$y^3-2y^2+2y^2-y^3$

$=y^3-y^3+2y^2-2y^2$

=0

∴ (x-y) is a factor Answer.

17. Solve the equation Log_{10} (2m+6)= 1+ Log_{10} (m-1)

Solution

$\text{Log}_{10}(2m+6)=1+\text{Log}_{10}(m-1)$

$\text{Log}_{10}(2m+6)= \text{Log}_{10}10+\text{Log}_{10} (m-1)$

$\text{Log}_{10} (2m+6)=\text{Log}_{10}(10\times(m-1))$

Take antilogs on both sides

2m+6=10m-10

2m-10m=-6-10

-8m=-16 (Divide both sides by -8)

∴m=2

Checking

If m=2, then LHS becomes

$\text{Log}_{10}(2\times2+6)=\text{Log}_{10}(4+6)$

$=\text{Log}_{10} 10=1$

Also RHS becomes $1+\text{Log}_{10} (2-1)$

$=1+\text{Log}_{10} 1$

$\text{Log}_{10} 1$ is 0, so RHS remains 1.

∴LHS=RHS=1 as required.

18. Figure 6, shows a rectangular prism, in which BC=3cm, EF=4cm, BF=6cm and angle ABC=90^0. Calculate the volume of the prism.

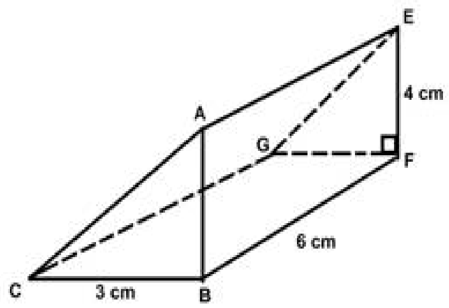


Figure 6

Solution

Volume=Area of cross section×Length

$=\left(\frac{1}{2}\times 3\times 4\right)\text{cm}^2\times 6\text{cm}$

$=(3\times2)\text{cm}^2\times6$ cm

$=(6\times6)$ cm^3

=36cm^3 Answer.

19. Given that the universal set ξ ={11,14,15,17,18,20,23,26}, Set **X**={11,14,15,17,18,20}, and set **Y**={15,17,18,20,23,26}, find **X' ∪ Y'**.

Solution

X'={23,26}

Y'={11,14}

∴X' ∪ Y'

={11,14,23,26} Answer

20. In a plastic bag there are x blue pens, 6 black pens and 4 red pens. If the probability of picking a red pen is $\frac{1}{5}$, calculate the number of blue pens.

Solution

P(red)=$\frac{4}{x+6+4}$

$\frac{1}{5}=\frac{4}{x+10}$ (By observation x should be 10 so that RHS becomes $\frac{1}{5}$)

$\frac{1}{5}=\frac{4}{20}$

∴ x=10 blue pens Answer.

21. Solve the simultaneous equations $y=x^2$

y=5x-6

Solution

$y=x^2$

y=5x-6

Substitution of y in the second equation by x^2

i.e. $x^2=5x-6$

Equate $x^2=5x-6$ to 0,

i.e. LHS is 0.

$x^2-5x+6=0$

(x-2) (x-3)=0

X is 2 or 3.

When x is 2, $y=(2)^2=4$

When x is 3, $y=(3)^2=9$

Roots of the equation are:

x= (2,3)

y=(4,9) Answer.

2005 MALAWI SCHOOL CERTIFICATE OF EDUCATION EXAMINATION

MATHEMATICS SOLUTIONS

1. Factorise completely $3t^2$-4t+1.

   Solution

   $3t^2$-4t+1

   =$3t^2$-3t-t+1

   =3t (t-1)-1(t-1)

   = (t-1)(3t-1) Answer

2. Simplify $\sqrt{125}+\sqrt{5}-\sqrt{45}$ leaving your answer in surd form.

   Solution

   $\sqrt{125}+\sqrt{5}-\sqrt{45}$

   $=\sqrt{25\times 5}+\sqrt{5}-\sqrt{9\times 5}$

   $=5\sqrt{5}+\sqrt{5}-3\sqrt{5}$

   $=6\sqrt{5}-3\sqrt{5}$

   $=3\sqrt{5}$ Answer

3. In figure 1, TP is a tangent to the circle APB at P and AB//PT. Prove that AP=BP

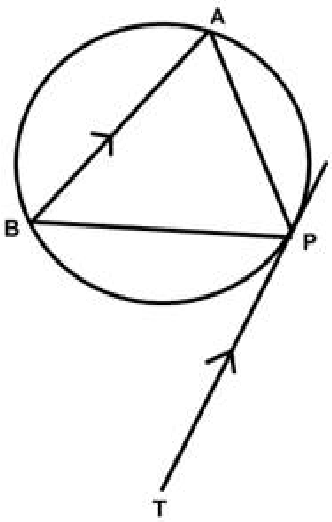


   Figure 1

   Solution

   Angle BPT=Angle ABP (alternate angles AB//PT)

   Also angle BPT=Angle BAP (angles in alternate segments)

   ∴ Angle ABP=Angle BAP

   ∴ Triangle ABP is isosceles.

   ∴ AP=BA(as required)

4. Simplify $\frac{x^2-y^2}{x^2-xy}$

   Solution

   $\frac{x^2-y^2 \text{(difference of 2 squares)}}{x^2-xy}$

   $=\frac{(x+y)\cancel{(x-y)}}{x\cancel{(x-y)}}$

   $=\frac{x+y}{x}$ Answer

5. Given that {a, c, e, h, i, m,s} find n(Y').

   Solution

   ξ =(a,c,e,h,I,l,m,s,t,w}

   y= {a,c,e,h,i,m,s}

   y'={l,t,w}

   ∴n(y') =3 Answer.

6. The function y=2+x has the range {3,6} Find its domain.

   Solution

   To find domain for range 3,

   3=2+x

   3-2=x

   1=x

   ∴ x=1

   For range 6

   6=2+x

   6-2=x

   4=x

   ∴ x=4

   ∴ Domain={1,4} Answer

7. Given that Log_m 27=3, find m

   Solution

   $\text{Log}_m 27=3$

   Change to exponential equation

   27=m3

   $3^3=m^3$(Equate bases)

   3=m

   m=3 Answer.

8. Make r the subject of the formula $S=\pi\,(2r)^2$

   Solution

   $S=\pi\,(2r)^2$(divide both sides by π)

   $\frac{S}{\pi}=\pi\times(2r)^2\times\frac{1}{\pi}$

$\frac{S}{\pi} = 2r \times 2r$

$\frac{S}{\pi} = 4r^2$

$\sqrt{\frac{S}{4\pi}} = \sqrt{r^2}$ **Answer**

9. In figure 2, PQR is a triangle such that PQ=6 cm, QR =10cm and RP=7cm. Calculate angle PRQ to the nearest degree.

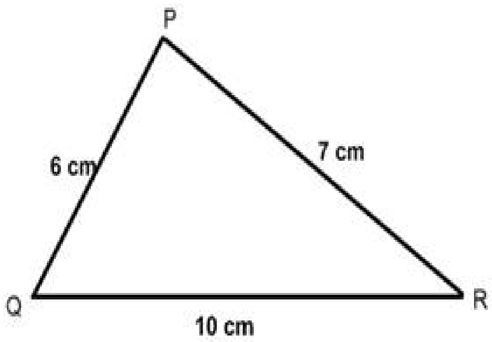


Figure 2

Solution

(By cosine rule)

COS R=$\frac{p^2+q^2-r^2}{2pq}$

$=\frac{10^2\times9^2\times6^2}{2\times10\times7}$

$=\frac{100+49-36}{140}$

$=\frac{149-36}{140}$

$=\frac{113}{140}$

=0.807142857

Cos $^{-1}$=36^0 Answer

10. A trapezium has a height of 3 cm and its area is 6 cm^2. Calculate the area of a similar trapezium with a height 12 cm.

Solution

Area of trapezium=$\frac{1}{2}$(Sum of // Sides) × height. But these are similar shapes whose area is in the squares of ratio of corresponding sides.

i.e. $\frac{(12)^2}{(3)^2} = \frac{a\ cm^2}{6cm^2}$

9 cm^2=144×6 cm^2.

9cm^2= $\frac{144\times6}{9}$

= 90 cm^2 Answer

11. Find the equation of a straight line passing through the point (0, 7) and line y=2x+5.

Solution

Gradient of the line

y=2x+5 is co-efficient of x which is z.

Parallel lines have the same gradients.

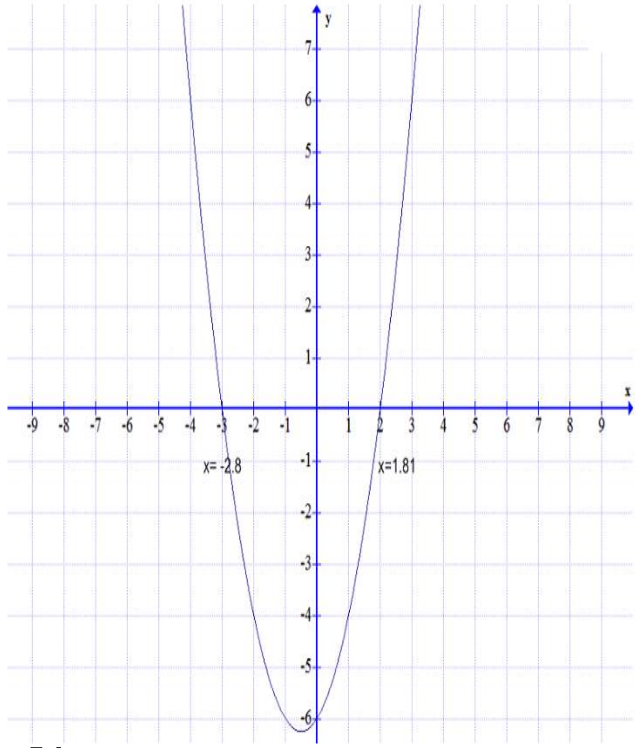
∴ A straight line passing through (0, 7) has gradient 2.

Equation is y-y1=m(x-x1) where **m** is gradient.

By substitution

y-7=2(x-0)

y-7=2x-0



y=2x+7 Answer.

12. Given that **P** varies as a product of **q** and **r2**, and that **p=50** when **q=1** and **r=5,** find **P** when **q=3** and **r=8.**

Solution

Pα qr^2

P=kqr^2, K is constant.

50= k×1×5×5

50=25k (Divide each term by 25)

K=2

∴p=2qr^2

Where q=3 and r=8

Age(Yrs)	Deviation from mean	Square of deviation
10	-2.6	6.76
11	-1.6	2.56
13	0.4	0.16
14		
15	2.4	5.76
Total	0	

P=2×3×8×8

=384 Answer

13. A farmer is selling at most 70 chickens out of which less than 30 are hens. Using x to represent the number of hens and y to represent the number of cocks, write down four inequalities involving x and y.

Solution

If x=hens and y=cocks

Then x+y≤ 70,

x<30

y>x and

y> 40 are the inequalities Answer.

14. Figure 3, shows a graph of **$y=x^2+x-6$**

Solution

$y=x^2+x-6$

$x^2=5-x$,

$x^2+x-5=0$.

To bring x^2+x-5 to its original form, add -1 o both sides

i.e. $x^2+x-5-1=-1$

x+x-6=-1

y=-1 is a parallel line that passes through the y-axis at -1, parallel to the x-axis. This will cut the curved graph (parabola) at 2 points. From these two points, draw dotted lines to meet the x-axis at -2.8 and 1.8 respectively.

∴x=-2.8 and 1.8 Answer.

Figure 3

15. The table below shows ages of 5 pupils with the mean age of 12.6 years.

Table 1

Solution

Deviations from mean should have a sum of 0. i.e. positive values + negative values=0

-2.6+ -1.6 +0.4 +2.4+ Deviating from mean up against 14 years.

-4.2+2.8=0

So we add 1.4 to 2.28 to make of =0 on LHS.

1.42=1.96

Variance=(6.76+2.56+0.16+1.96+5.76)÷5

=17.20÷5

=3.44 Answer.

16. A chord of a circle centre O is 8.4cm long. If the radius of the circle is 7cm long, sketch the diagram calculate the distance of the chord from the centre of the circle.

Solution

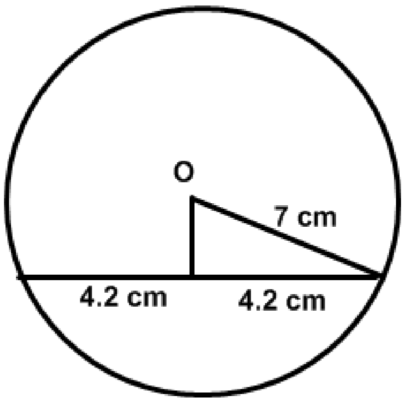


Figure 4

Distance of chord is OD.

$OD^2=(7)^2-(4.2)^2$

$=49cm^2-17.64cm^2$

$=\sqrt{31.36\ cm2}$

=5.6 cm Answer.

17. The nth term of an Arithmetic progression is 5n-3. Calculate the sum of the first 6 terms of the AP.

Solution

The terms are

$1^{st}=5\times1-3=2$

$2^{nd}=5\times2-3=7$

$3^{rd}=5\times3-3=12$

$4^{th}=5\times4-3=17$

$5^{th}=5\times5-3=22$

$6^{th}=5\times6-3=27$

Sum=2+7+12+17+22+27

=87 Answer

18. Given that $(4x^2-9)(Bx+C)$ is identical to $16x^3+24x^2-36x-54$, calculate values of B and C given that they are all positive.

Solution

Expand $(4x^2-9)$ by (Bx+C)

$=(4x^2-9)\times(Bx+C)$

$=4Bx^3+4cx^2-9Bx-9c$

$=4Bx^3+4Cx^2-9Bx-9c$

$=16x^3+24x^2-36x-54$

=4B=16

$=B=\frac{16}{4}=4$

4C=24

$C=\frac{24}{4}=6$

∴B=4 and C=6 Answer

19. Figure 5 is a tree diagram illustrating the probability of a student passing Agriculture and Geography in an examination. The probability of passing Geography in the examination is $\frac{7}{10}$, and the probability of passing Agriculture after one has passed Geography is $\frac{5}{7}$. The probability of passing Agriculture after one has failed Geography is $\frac{1}{6}$. Calculate the probability of a student passing Agriculture.

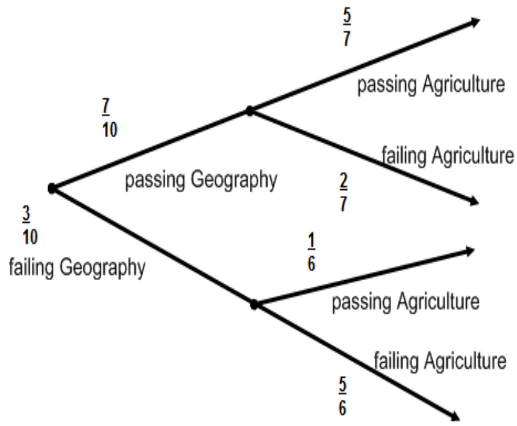


Figure 5

Solution

Probability of passing Agriculture

$=\left(\frac{7}{10}\times\frac{5}{7}\right)+\left(\frac{3}{10}\times\frac{1}{6}\right)$

$=\frac{1}{2}+\frac{1}{20}$

$=\frac{10+1}{20}$

$=\frac{11}{20}$ **Answer**

20. Given that $\overrightarrow{AB}=\begin{pmatrix}-9\\4\end{pmatrix}$, calculate the length of $\overrightarrow{AB}$ leaving your answer correct to 3 significant figures.

Solution

Length$_{\overrightarrow{AB}}$ $=\sqrt{(-9)^2+(4)^2}$

$=\sqrt{97}$

=9.85 Answer

21. Figure 6 shows a right-cone whose vertical angle BAC=1160, the diameter of its base BC=176mm and AX is its height. Calculate the length of AX.

Solution

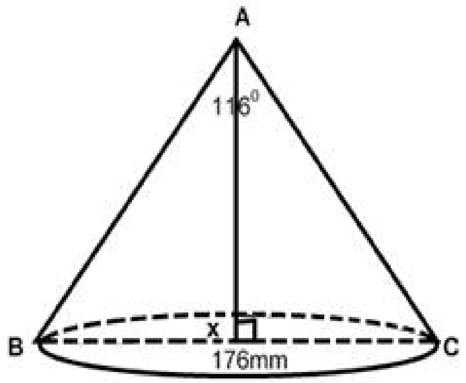


Figure 6

The height AX is perpendicular to the diameter. It cuts diameter into two equal parts, (88cm) similarly, it divides the vertical angle in two equal halves.(bisects the vertical angle 58^0)

Triangle AXC is right-angled at X.

$\therefore \frac{88cm}{AX}$=Tan 58^0.

AX=$\frac{88cm}{Tan\ 58^0}$

=55 cm Answer.

22. **Solve the simultaneous equations.**

xy= -9

y=x+6

Solution

xy= -9

y= x+6

x(x+6)=-9.

$x^2+6x+9=0$

(x+3) (x+3)=0

x=-3 twice

-3×y=-9

-3y=-9

y= 3 twice

∴ Roots are (x=-3, -3) (y=3,3) Answer

2006 MALAWI SCHOOL CERTIFICATE OF EDUCATION EXAMINATION

MATHEMATICS SOLUTIONS

1. Simplify $\frac{3x+6}{(x-1)(x+2)}$

Solution

=$\frac{3(x+2)}{(x-1)(x+2)}$

=$\frac{3}{x-1}$ **Answer**

2. Factorise completely

$2x^2+4xy-30y^2$

Solution

$2x^2+4xy-30y^2$

$2x^2 \times -30y^2=-60x^2y^2$

(factors are 10xy-6xy)

$=2x^2+10xy-6xy-30y^2$

=2x(x+5y)-6y(x+5y)

=(x+5y)(2x-6y)

=2(x+5y)(x-3y) Answer.

3. A cuboid is 76 cm long, 50 cm wide and 40 cm high. Calculate the volume of the cuboid.

Solution

Volume=(l×w×h)

=76cm×50cm×40cm

=152,000cm3 Answer

4. **If $f(x)=8^x-6$, find $f\left(\frac{2}{3}\right)$**

Solution

$f(x)= 8^x-6$

$f\left(\frac{2}{3}\right)=8^{\frac{2}{3}}-6$

$=\sqrt[3]{(8)^2}$ -6

=4-6

=-2 Answer

5. Figure 1 is a triangle ABC in which angle BAC=600, AC=10cm and the area of the triangle is $15\sqrt{16cm^2}$

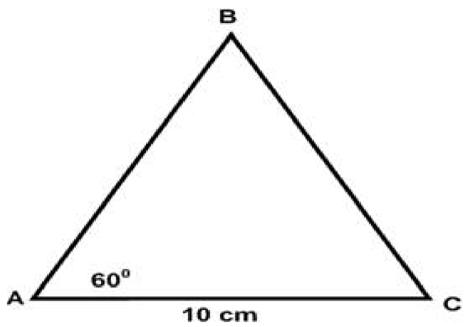


Figure 1

Calculate the length of AB leaving the answer in its simplest surd form.

Solution

$\frac{x}{5\text{ cm}}$ **=Sin 60^0**

x =5× Sin 60^0.

6. Table 1 shows marks that student **A** and student **B** got from test. Student **A** sat for 5 tests while student **B** sat for 4 test. Student **B** has mark x missing.

Table 1

STUDENT A	55	70	80	30	65
STUDENT B	67	60	x	53	

Given that the mean mark of student A is the same as the mean mark of student B, calculate the value of x.

Solution

Mean of student A

=(55+70+80+30+65)÷5

=300÷5

=60

Mean of student B

=(67+60+53+x)÷4

$=-45+\frac{x}{4}$

$=\frac{x}{4}+\frac{45}{7}=60$

X+180=240

∴x=240-180

x=60 Answer

6. John is twice as old as Mary. If the sum of the squares of their ages is 125. How old is Mary?

Solution

Let Mary be x years

Let John be 2x

Squares of their ages are$(x)^2$, $(2x)^2$

$(x)^2+(2x)^2=125$

$x^2+4x^2=125$

$\frac{5x^2}{5}=125$

$x^2=5$

Mary is 5 years old.

7. Without using a calculator or four-figure tables, find Cos B if Sin B=0.8.

Solution

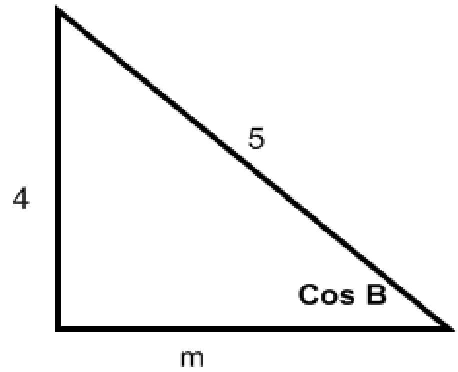


Figure 2

Sin B=0.8/$\frac{8}{10}$/$\frac{4}{5}$

$m^2=5^2-4^2$

=25-16

=9

$\therefore\sqrt{m^2}=\sqrt{9}$

$\therefore$Cos 23=$\frac{3}{5}$

=0.6 Answer.

8. Triangle ABC has vertices A(-1,2), B(3,3) and C(2,-1). Prove that angle BAC=angle ACB.

Solution

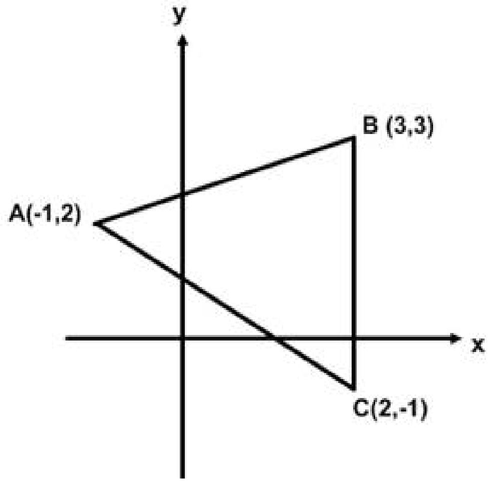


Figure 3

First, find lengths of sides BA and BC.

BA=$\sqrt{(y^2-y_1)^2+(x_2-x_1)\ \ ^2}$

=$\sqrt{(2-3)^2+(-1-3)^2}$

=$\sqrt{(-1)^2+(-4)^2}$

=$\sqrt{1+16}$

=$\sqrt{17}$

=4.12

Also BC=$\sqrt{(-1-3)^2+(2-3)^2}$

=$\sqrt{(-4)^2+(-1)^2}$

=$\sqrt{16+1}$

=4.12

∴ Triangle ABC is isosceles

∴ Angle BAC=Angle ACB(base angles)

9. Find the values of x and y in the following matrix equation.

Solution

$\begin{pmatrix}6 & 3\\ 2 & -3\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}12\\ -4\end{pmatrix}$

Working

$\begin{pmatrix}6 & 3\\ 2 & -3\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}12\\ -4\end{pmatrix}$

$\begin{pmatrix}6\times x+3\times y\\ 2\times x-3\times y\end{pmatrix}=\begin{pmatrix}12\\ -4\end{pmatrix}$

6x+3y=12

2x-3y=-4

8x = 8

x= 1

6+3y=12

3y=6

y=2

∴ x=1 and y=2 Answer

13. Given that x α $\frac{y}{z}$. When x= 10, y=2 and z=4. Find value of x when y=1 and z=5.

Solution

x α $\frac{y}{z}$.

x= $\frac{Ky}{z}$ (where k is constant)

10=$\frac{K\times 2}{4}$

10=$\frac{2k}{4}$

40=2K

20=k

∴ k=20

Thus x=$\frac{20y}{z}$

Where y=1 and z=-5

X=$\frac{20\times 1}{5}$

X= 4 Answer.

14. Two lines **G** and **H** intersect at a point **P**. G passes through the point (-4,0) and (0,6). Given that **H** has the equation: y=4x-4, find by calculation, the coordinates of P.

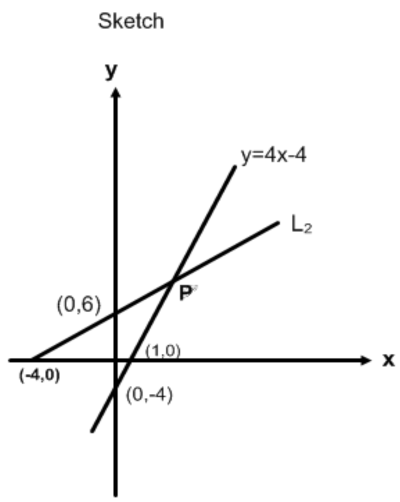


Figure 4

Gradient for line 2(L_2)

=$\frac{6-0}{0-(-4)}$

=$\frac{6}{4}$

=$\frac{3}{2}$

Equation of line 2 is y_2y_1=m(x-x_1)

=y-0=$\frac{3}{2}$(x-(-4)

$y-0=\frac{3}{2}x+6$

$$\begin{Bmatrix} y=\frac{3}{2}x+6 \\ y=4x-4 \end{Bmatrix}\times 2$$

2y=3x+12 (i)

2y=8x-8 (ii)

Subtract (i) from (ii)

5x=20,

x= 4

Coordinates of p

2y= 3×4+12,

2y=24,

y=12.

15. Figure 5 shows a venn diagram representing set of all numbers(N), set of even numbers(E), set of odd numbers (O) and set of prime numbers (P). Copy the venn diagram and place the numbers $\frac{1}{2}$, 2,6,9 and 13 in the right places.

Solution

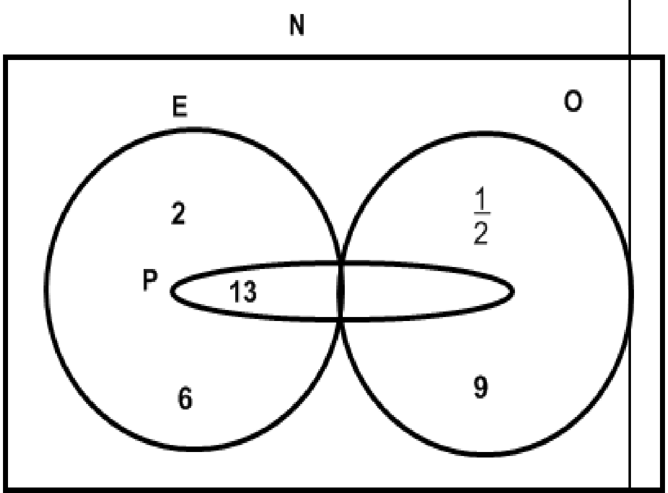


Figure 5

Answer

16. Using a ruler and a pair of compasses only, draw a line AB=10cm, and construct a circle with the line AB as a diameter. Mark a point C on the circle such that AC= 6cm. Join AC and BC. Construct a tangent CP such that angle BCP is a cute.

Working

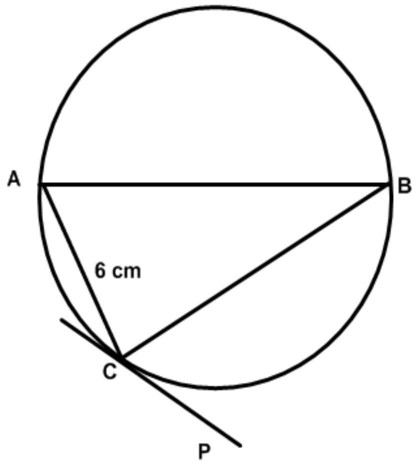


Figure 6

16. Figure 7, shows graphs of x+y=8 and x-2y+4=0. Copy the figure on the graph paper provided and show the region by the following inequalities.

x≥0, x+y≤8, x-2y+4≥0 by shading the unwanted region.

Working

For x≥0, Boundary line on the sketch is x=0, (solid line)

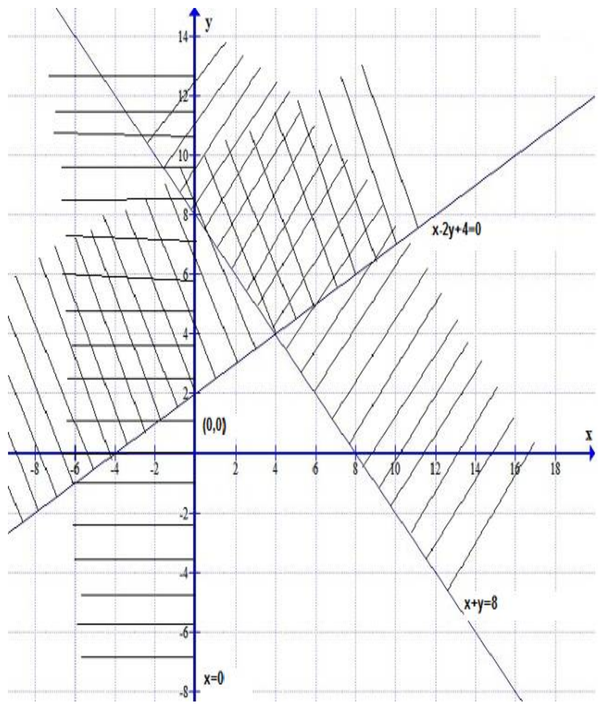


Figure 7

Shading off behind vertical axis.
For $x+y\leq 8$, Boundary line is $x+y=8$, (solid line)
Shading off above BL.
$x-2y+4\geq 0$, Boundary line is $x-2y=-4$, (solid line)
(y subtract 1st $-2y\geq -x-4$)
$y\leq \frac{x}{2}+2$ (symbol reversed)
$\mathbf{y=\frac{x}{2}+2}$

17. Figure 8 shows a circle ABCD in which AB and DC are produced to G and F respectively such that GF is parallel to AD.
Prove that Quadrilateral GBCF is cyclic.

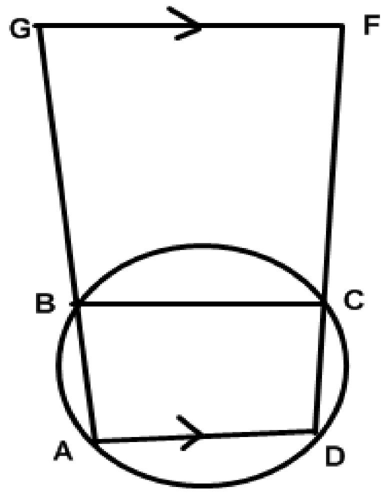


Figure 8
Solution
ABCD is cyclic quadrilateral(vertices lie on circumference of a circle ABCD)
$a+b=180^0$ (allied angles GF//AD)
$a=a_1$ (Exterior angle of cyclic quadrilateral ABCD is equal to opposite interior angle)
$\therefore a_1+b$ in quad GBCF$=180^0$
Also $c+d=180^0$ (allied angles GF//AD)
$c=c_1$(exterior angle of a cyclic quadrilateral)
$\therefore c_1+d=180^0$.
∴Quadrilateral GBCF is cyclic quad as required (two pairs of opposite angles are supplementary-add up to 180^0)
19. Given that *$Log_{10}n-Log_{10}m=2\ Log_{10}h$*, show that *$n=mh^2$*.
Show that $n=mh^2$.

Solution
$Log_{10}n-Log_{10}m=2\ Log_{10}h$(given)
Thus $Log_{10}\left(\frac{n}{m}\right)= Log_{10}h^2$(Laws of indices)
Take antilog on both sides.
$\frac{n}{m}=h^2$(make **n** subject. Multiple both sides by **m**)
$\frac{n}{\not m}\times \not m = m\times h^2$
$\therefore n=mh^2$ as required.
18. Coin A is tossed followed by coin B. The probability that coin A shows head is $\frac{1}{2}$ while the probability that coin B shows head is $\frac{1}{4}$.
Using a tree diagram, calculate the probability that both coins A and B show tails.

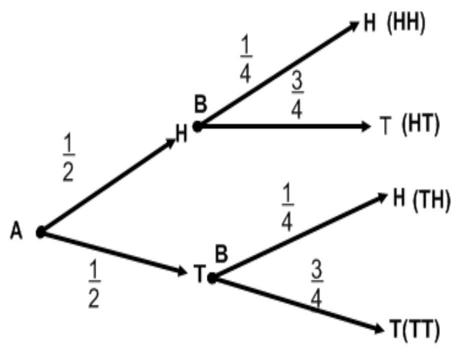


Figure 9
Solution
A and B both shows tails from one branch TT.
These are independent events so PGT$=\frac{1}{2}\times\frac{3}{4}$
$=\frac{3}{8}$ **Answer**
19. Find the sum of the first 12 terms of the following GP.
$\frac{1}{2187},\frac{1}{729},\frac{1}{243},\ldots\ldots$ Answer to two decimal places.
Solution
Common ratio$=\frac{1}{729}\div\frac{1}{2187}$
$=\frac{1}{729}\times\frac{2187}{1}$
$=3$
Sum $=\frac{\frac{1}{2187}(3^{12}-1)}{3-1}$
$=\left\{\frac{1}{2187}(531441-1)\right\}\div 2$

$=\left\{\frac{1}{2187}\times 531440\times\frac{1}{2}\right\}$

=121.4977

=121.50 to 2 decimal places Answer

20. Simplify $\frac{x^2-2}{x-\sqrt{2}}$

Solution

$\frac{x^2-2}{x-\sqrt{2}}$

$=\frac{(x+\sqrt{2})\ \cancel{(x-\sqrt{2})}}{\cancel{(x-\sqrt{2})}}$

=$(x+\sqrt{2})$ Answer

2007 MALAWI SCHOOL CERTIFICATE OF EDUCATION EXAMINATION

MATHEMATICS SOLUTIONS

1. **Express $\frac{\sqrt{2}}{\sqrt{3}}$ with a rational denominator**

   Solution

   $\frac{\sqrt{2}}{\sqrt{3}}$ (Multiply both Numerator and denominator by $\sqrt{3}$)

   $=\frac{\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

   $=\frac{\sqrt{2\times3}}{\sqrt{3\times3}}$

   $=\frac{\sqrt{6}}{\sqrt{9}}$

   $=\frac{\sqrt{6}}{3}$ **Answer**

2. **Given that $\underline{a}=\begin{pmatrix}-2\\-4\end{pmatrix}$ and $\underline{b}=\begin{pmatrix}-4\\0\end{pmatrix}$, find $\frac{1}{2}(\underline{b}-\underline{a})$**

   Solution

   $\frac{1}{2}(\underline{b}-\underline{a})=\frac{1}{2}\left\{\begin{pmatrix}-4\\0\end{pmatrix}-\begin{pmatrix}-2\\-4\end{pmatrix}\right\}$

   $=\frac{1}{2}\begin{pmatrix}-4-(-2)\\0-(-4)\end{pmatrix}$

   $=\frac{1}{2}\begin{pmatrix}-4+2\\0+4\end{pmatrix}$

   $=\frac{1}{2}\begin{pmatrix}-2\\4\end{pmatrix}$

   $=\begin{pmatrix}\frac{1}{2}\times-2\\\frac{1}{2}\times4\end{pmatrix}$

   $=\begin{pmatrix}-1\\2\end{pmatrix}$ **Answer**

3. Factorise completely $2x^2-4x-126$

   Solution

   $2x^2-4x-126$(factor out 2, a common factor)

   $2(x^2-2x-63))$

   Multiply x^2 by $-63x^2$ (factors are $-9x + 7x$)

   $=2(x^2-9x+7x-63)$

   $=2\{x(x-9)+7(x-9)\}$

   $=2\{(x-9)(x+7)\}$

   $=2(x-9)(x+7)$ Answer

4. The fraction $f(x)=\frac{1}{3x-1}$. Given that $\{-1,0,2\}$is the domain, find the range.

   Working

   $f(x)=\frac{1}{3x-1}$

   Domain={-1,0,2}

   By substitution

   $\therefore$ Range$=\left\{\frac{1}{3\times(-1)-1},\frac{1}{3\times(0)\times-1},\frac{1}{3\times(2)-1}\right\}$

   $=\left\{\frac{1}{-3-1},\frac{1}{0-1},\frac{1}{6-1}\right\}$

   $=\left\{\frac{1}{-4},\frac{1}{-1},\frac{1}{5}\right\}$

   $=\left\{-\frac{1}{4},-1,\frac{1}{5}\right\}$ **Answer**

5. Express b in terms of a and c in the formula $c=ab-\frac{b}{a}$

   Solution

   $C=ab-\frac{b}{a}$

   (Multiply each term by a)

   $ac=a^2b-b$

   $\therefore a^2b-b=ac$

   Factor out b on LHS.

   $b(a^2-1)$ ac (divide both sides by a^2-1)

   $b=\frac{ac}{a^2-1}$

   or

   $b=\frac{ac}{(a+1)(a-1)}$ **Answer**

6. Figure 1 shows a tangent to a circle ABC with centre O. Line CT is parallel to BA and angle ATC=32°. Calculate angle ACB.

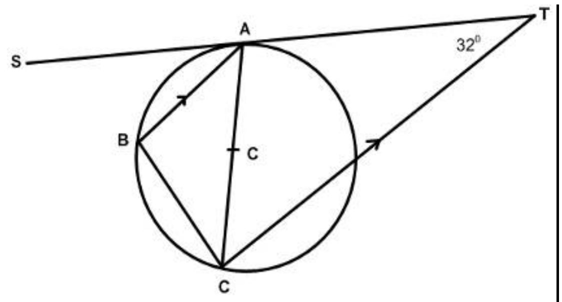


Figure 1

Solution

In triangle TAC, angle TAC=90^0(radius OA perpendicular to tangent TA)

Angle ATC=32^0(given)

∴ Angle TAC=180^0-(90^0+32^0) (sum angle in a Triangle)

=180^0-122^0.

=58^0.

Angle TCA=Angle CAB, (alternate angles CT//BA)

But angle ABC=90^0 (angle in a semicircle)

∴ Angle ACB=180^0-(90^0+58^0) (angle sum in a triangle ABC)

=180^0-148^0

=32^0 Answer.

7. The gradient of a straight line passing through point p(-2,5) is -$\frac{1}{2}$. Find the equation of the line in the form **y=mx+c.**

Solution

Equation of the line is $y-y_1=m(x-x_1)$ where

y_1 is first y coordinate(5)

x_1 is first x coordinate (-2) and m is gradient.

By substitution, it yields

$y-5=-\frac{1}{2}(x-(-2))$

$y-5=-\frac{1}{2}(x+2)$

$y-5=-\frac{x}{2}-1$

$y=-\frac{x}{2}-1+5$

∴$y=-\frac{x}{2}+4$ Answer

8. Figure 2 is a venn diagram showing the number of elements in sets R, S and universal set ξ

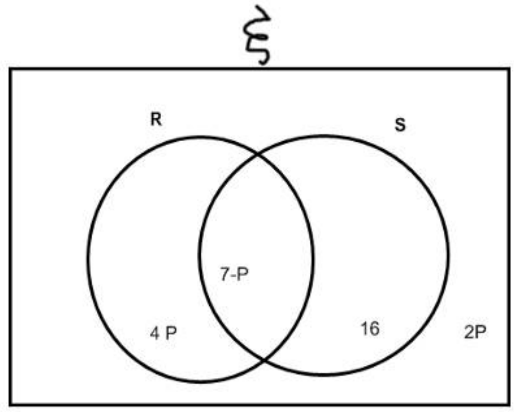


Figure 2

If n(RUS)=29, calculate the value of p.

Solution

n(RUS)=29

(4p+7-p+16)=29

(4p-p+7+16)=29

3p+23=29

3p=29-23

3p=6

P=2

∴ p is 2 Answer

9. Given that A=$\begin{bmatrix}2 & 6\\1 & -4\end{bmatrix}$, B=$\begin{bmatrix}-3 & 2\\0 & -5\end{bmatrix}$ and C=$\begin{bmatrix}-1 & 8\\-5 & 7\end{bmatrix}$, simplify $\frac{1}{4}(A-B+C)$

Solution

$\frac{1}{4}(A-B+C)$

$\frac{1}{4}\left\{\begin{pmatrix}2 & 6\\1 & -4\end{pmatrix}-\begin{pmatrix}-3 & 2\\0 & -5\end{pmatrix}+\begin{pmatrix}-1 & 8\\-5 & 7\end{pmatrix}\right\}$

$=\frac{1}{4}\left\{\begin{pmatrix}2-(-3) & 6-2\\1-0 & -4-(-5)\end{pmatrix}+\begin{pmatrix}-1 & 8\\-5 & 7\end{pmatrix}\right\}$

$=\frac{1}{4}\left\{\begin{pmatrix}2+3 & 6-2\\1-0 & -4+5\end{pmatrix}+\begin{pmatrix}-1 & 8\\-5 & 7\end{pmatrix}\right\}$

$=\frac{1}{4}\left\{\begin{pmatrix}5 & 4\\1 & 1\end{pmatrix}+\begin{pmatrix}-1 & 8\\-5 & 7\end{pmatrix}\right\}$

$=\frac{1}{4}\begin{Bmatrix}5+(-1) & 4+8\\1+(-5) & 1+7\end{Bmatrix}$

$=\frac{1}{4}\begin{Bmatrix}4 & 12\\-4 & 8\end{Bmatrix}$

$=\begin{pmatrix}\frac{1}{4}\times 4 & \frac{1}{4}\times 12\\\frac{1}{4}\times(-4) & \frac{1}{4}\times 8\end{pmatrix}$

$=\begin{pmatrix}1 & 3\\-1 & 2\end{pmatrix}$ Answer

10. Simplify $\frac{1}{a-b}+\frac{1}{a+b}$

Solution

$\frac{1}{a-b}+\frac{1}{a+b}$ (common denominator is (a-b)(a+b)

$=\frac{(a+b)+(a-b)}{(a-b)(a+b)}$

$=\frac{2a}{(a-b)(a+b)}$ **Answer**

11. Given that $Log_a2=0.668$ and $Log_a3=0.884$, evaluate Log_a12.

 Solution

$Log_a12=Log_a(2\times2\times3)$

$=Log_a(2^2\times3)$

$= Log_a2^2+Log_a3$

$=2\times Log_a2+ Log_a3$

$=(2\times0.668)+0.884$

$=1.336+0.884$

=2.22 Answer

Or $Log_a(2\times2\times3)$

$=Log_a2+Log_a2+Log_a3$

$=0.668+0.668+0.884$

=2.22 Answer

12. P varies directly as x^3 and inversely as y. When x=2 and y=4, P=3. Find the value of x when P=12 and y=4.

Solution

$P\alpha\, x\frac{x^3}{y}$

$P\ \alpha\ \frac{Kx^3}{y}$, where k is a constant.

$3=\frac{K\times2^3}{4}$

$3=\frac{8K}{4}$

$\frac{3}{2}=\frac{2K}{2}$

$K=\frac{3}{2}$

$\therefore P=\frac{3x^3}{2y}$

$12=\frac{3\times x^3}{2\times4}$

$12\times2\times4=3x^3$

$\frac{12\times2\times4}{3}=x^3$

$4\times2\times4=x^3$

$x^3=32$

$x=\sqrt[3]{32}$

=3.2 Answer

13. When the polynomial x^3+5x^3+Kx+3 is divided by(x+2) it gives a remainder of 1. Find the value of K.

Solution

Let x+2=0

x=-2

Substitute x by -2 in the polynomial

x^3+5x^2+Kx+3

$=(-2)^3+5(-2)^2+K(-2)+3$

$= -8+20-2K+3$

$=23-8-2K$

$=15-2K$

$1=15-2K$

Thus $-2K+15=1$

$-2K=1-15$

$\frac{-2K}{-2}=\frac{-14}{-2}$

K=7 Answer

14. The fourth term of an Arithmetic progression is 11 and the seventh term is 20. Calculate the first term.

Solution

n^{th} term is a+(n-1)d where a is first term, d is common difference and n is number of terms.

Forth term is a+(4-1)d=11

a+3d=11-1

seventh term is a+(7-1)d=20

a+6d=20-2

solve the equation 1 and 2 simultaneously

a+3d=11

a+6d=20-2

subtract 1 from 2

3d=9

d=3

solve for a by substituting d by 3 in equation 1.

a+(3×3)=11

a+9=11

a=11-9

a=2

∴The first term is 2. Answer

15. Solve the simultaneous equations

 y=x+2

$x^2-xy=4$

Solution

y=x+2 (substitute y by x+2 in the second equation)

x^2-xy=4

x^2-x(x+2)=4

x^2-x^2-2x=4

-2x=4

x=-2

when x is -2, y=-2+2

y=0

∴ x=-2 and y=0 Answer.

16. Figure 3 shows the region R bounded by three inequalities.

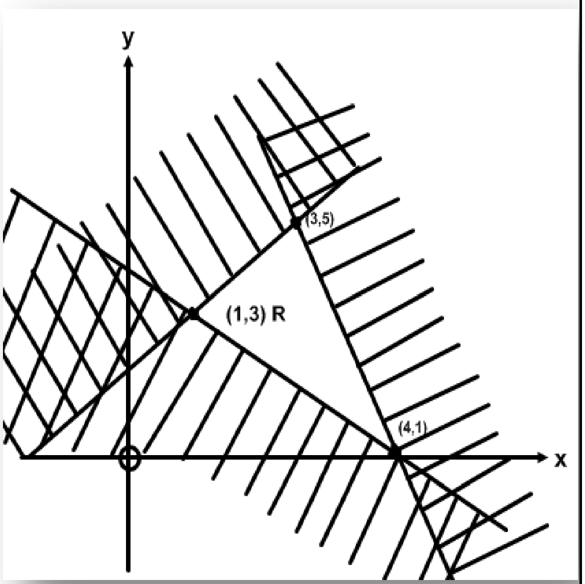


Figure 3

Calculate the maximum value of 5x-4y+8 in this region

Solution

At (4,1) the value is

5×4-4×1+8

=20+8-4

=24

At (3,5) the value is

5×3-4×5+8

=15-20+8

=23-20

=3

At (1, 3R) the value is 5×1-4×3R+8

=4+8-12R

=8-12R Answer

∴ The maximum value cannot be said between 24 and 8-12R because it is not known what R's value is. Suppose it is -2, it would bring

8-12×(-2)

=8+24

=32

17. Calculate the total surface area of a solid hemisphere of radius 21cm. **(Area of a sphere=$4\pi r^2$;Take $\frac{22}{7}\pi$)**

Solution

Area of a hemisphere

$=\frac{4\pi r^2}{2}$

2 3

$=\frac{4\times22\times\not{21}\times21}{\not{7}\times\not{2}}$

=2×22×3×21

=44×63

=2772 cm^2 Answer

18. Figure 4, shows the speed time graph of a moving object.

Use the graph to find the total distance travelled by the object in the first 35 seconds.

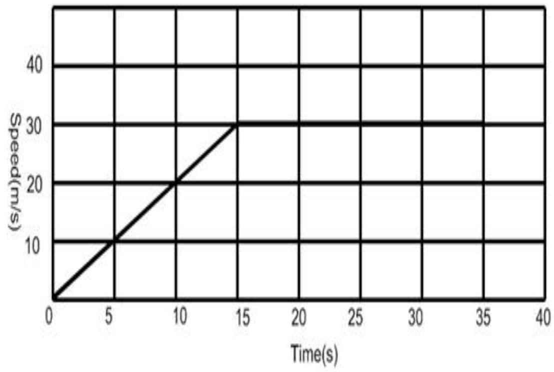


Figure 4

Solution

Distance covered in 1 second

$=\frac{\text{Vertical Interval}}{\text{Horizontal Interval}}$

$=\frac{10}{5}$

= 2 meters

In 15 seconds distance covered

=2×15 metres

=30 metres

The motion of an object between 15 seconds and 35 seconds is static/constant.

∴ No distance is covered because there is no speed.

Distance covered in 35 seconds

=30 metres+0

=30 metres. Answer

19. Figure 5 shows region G bounded by inequalities.

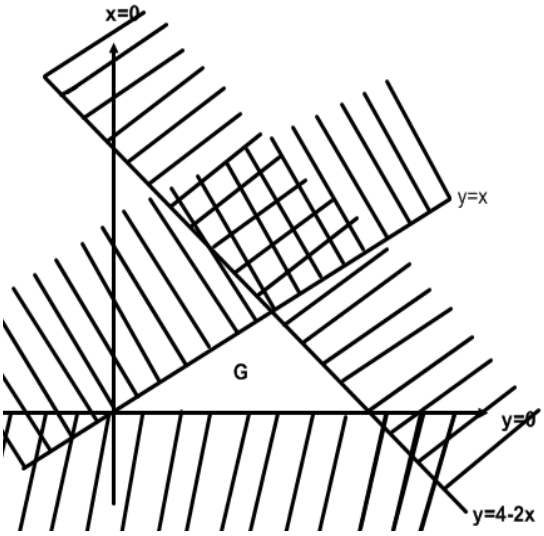


Figure 5

Solution

The inequalities are

(a) $y \geq 0$ (boundary line y=0 and line is solid)

(b) $y \leq x$ (boundary line y=x, line is solid)

(c) $y \leq 4-2x$(y=4-2x line solid) Answer.

20. The angle of depression of a car from the top of a pole is 35^0. If the top of the pole is 25m from the ground, calculate the distance of the car from the pole.

Solution

Sketch

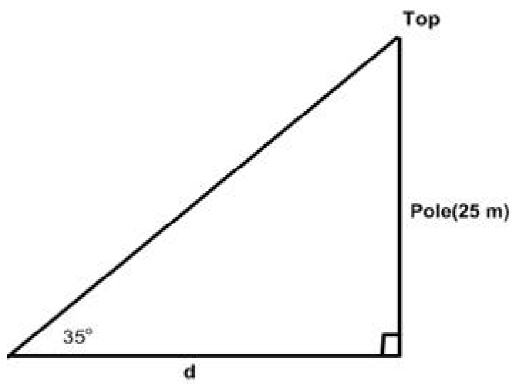


Figure 6

$\frac{25\text{m}}{\text{d}} = \frac{\text{Opposite}}{\text{Adjacent}} = \text{Tan } 35^0$

$\frac{25\text{m}}{\text{d}} = \text{Tan } 35^0$

$25\text{m} = \text{Tan } 35^0 \times \text{d}$

Or $\text{Tan } 35^0 \times \text{d} = 25\text{m}$

$\therefore \text{d} = \frac{25}{35^0}$

=36 m Answer

21. Three data values x,y and z have the following relationship.

$x=a^2-a$

$y=2-a$

$z=7+5a-a^2$

Calculate the mean of x, y and z in terms of a in its simplest form.

Solution

Mean=(x+y+z)÷3

$=a^2-a+2-a+7+5a-a^2$

$=a^2-a^2-a-a+2+7+5a$

=-2a+5a+9

= -2a+5a+9

=(3a+9)÷3

∴Mean=a+3 Answer

22. Triangle ABC is similar to triangle DBA. The area of triangle DBA is 24 cm^2, AB=8 cm and Db=4 cm.

Calculate the area of triangle ABC.

Solution

Triangles ABC and DBA are parallel (given)

$\therefore \frac{AB}{DB} = \frac{BC}{BA} = \frac{CA}{AD}$ (ratio of corresponding sides)

$\frac{8\text{cm}}{4\text{ cm}} = \frac{BC}{8\text{ cm}} = \frac{CA}{AD}$

Ratio of corresponding sides is

$\therefore \frac{8}{4} = \frac{2}{1}$

But are of similar triangles is in the ratio of squares of corresponding sides,

$=\frac{(2)^2}{(1)^2}$

$=\frac{2\times2}{1\times1}$

$=\frac{4}{1}$

$\therefore \frac{4}{1} = \frac{\Delta\,ABC}{\Delta\,DBA}$

$\frac{4}{1} = \frac{\Delta\,ABC}{24\text{cm}^2}$

$\therefore \Delta$ ABC= $(4\times24)\ \text{cm}^2$.

$=96\ \text{cm}^2$ Answer.

23. The probability of having an early lunch at a boarding school is $\frac{2}{3}$. When lunch is early, the probability of having beef is $\frac{7}{10}$ and when late, the probability of having beef is $\frac{1}{10}$ and when late, the probability of having beef is $\frac{1}{8}$. Draw a tree diagram to represent this information, completing all the branches.

Solution

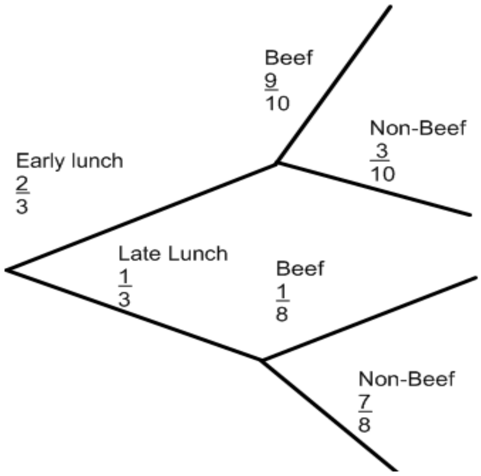


Figure 7

2008 MALAWI SCHOOL CERTIFICATE OF EDUCATION EXAMINATION

MATHEMATIC S SOLUTIONS

1. Factorise completely $3x^2y+5xy+2y$

   Solution

   $3x^2y+5xy+2y$ (Factor out y first)

   $y(3x^2+5x+2)$

   multiply $3x^2$ by $2=6x^2$ (Factors are $+3x+2x$)

   $=y(3x^2+3x+2x+2)$

   $=y\{3x(x+1)+2(x+1)\}$

   $=y(x+1)(3x+2)$ Answer

2. Without using a calculator or four figure-tables, simplify $\sqrt{27}\times\sqrt{32}$, leaving your answer in surd form.

   Working

   $\sqrt{27}\times\sqrt{32}$

   $=\sqrt{9\times3}\times\sqrt{16\times2}$

   $=\sqrt{9}\times\sqrt{3}\times\sqrt{16}\times\sqrt{2}$

   $=3\times4\times\sqrt{3}\times\sqrt{2}$

   $=12\sqrt{6}$ Answer

3. Simplify $\frac{x^2}{x}\times\frac{x^2}{x-1}$

   Solution

   $\frac{x^2}{x}\times\frac{x^2}{x-1}$

   X

   $=\frac{(x+1)\cancel{(x-1)}}{\cancel{x}}\times\frac{\cancel{x^2}}{\cancel{(x-1)}}$

   $=x(x+1)$ Answer

4. Make t the subject of the formula

   $M=K+\frac{3y^2}{t}$

   Solution

   $M=K+\frac{3y^2}{t}$ (Multiply each term/ fraction by the common denominator t)

   $t\times M=t\times K+\frac{t\times3y^2}{t}$

   $Mt=Kt+3y^2$ (subtract Kt from both sides)

   $t(M-K)=3y^2$

   (Divide both sides by M-K)

   $\therefore t=\frac{3y^2}{M-K}$ Answer

5. In figure 1 QN is a tangent to the circle LMN at N.

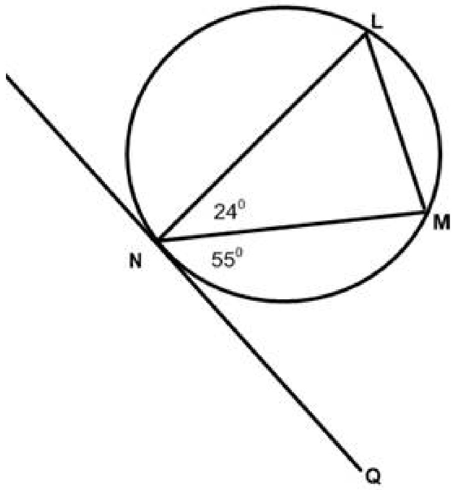


Figure 1

If angle $QNM=55^0$, angle $LNM=24^0$, calculate angle NML.

Working

Angle $LNR=24^0+55^0$

$=79^0$

$\therefore$Angle $N=180^0-79^0$

$=101^0$ (adjacent angles on straight line)
Angle N=Angle NML=101^0(angles in the alternate segments)

6. Given that $\text{Log}_x(\frac{1}{2})+\text{Log}_x16=3$, Find the value of x. Find the value of x.

Solution

$\text{Log}_x(\frac{1}{2})+\text{Log}_x16=3$

Recall rule of log

$\text{Log}_aMN=\text{Log}_aM+\text{Log}_aN$

∴ $\text{Log}_x(\frac{1}{2})+\text{Log}_x16=\text{Log}_x(\frac{1}{2}\times 16)$

$=\text{Log}_x8.$

Thus $\text{Log}_x8=3$

Change to exponential equation

$8=x^3$

$2^3=x^3$ (Evaluate bases)

∴x=2 Answer

7. Figure 2 shows an arrow diagram for the function (fx)=2x-5

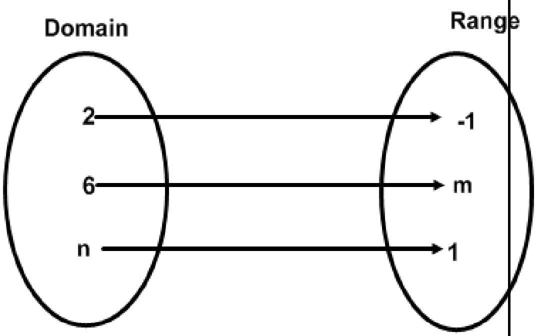


Figure 2

Calculate the values of *m* and *n*.

f(x)=2x-5

∴f(2)=2×2-5

=4-5

=-1 (This is the first range value)

∴ To find m, substitute f(6)

=2×6-5

=12-5

=7

∴To find n, 2x-5=1

2x=6

x=3

∴n=3 Answer

8. Find the sum of the first 20 terms of the arithmetic progressions 4,2,0........

Solution

A=4, n=20 and d=

Sn=$\frac{1}{2}$n$(2a+(n-1)d)$

=$\frac{1}{2}$×20$(2\times 4+(20-1)-2)$

=10×$(8+(19\times -2))$

=10×$(8+(-38))$

=10×(-30)

=-300 Answer

9. Given that y=2x-3 and y=(b-1)x+5 are graphs of two parallel straight lines, calculate the value of b.

Solution

y=2x-3 and y=(b-1)x+5 are parallel. (given)

∴ Gradients are equal.

Thus b-1=2

∴b=1+2

b=3 Answer

10. Solve the equation $3(a+1)^2-3=0$

Solution

$3(a+1)^2-3=0$

Expand $(a+1)^2$ i.e. (a+1) (a+1)

$3(a^2+2a+1)-3=0$

$3a^2+6a+3-3=0$

$3a^2+6a=0$

3a(a+2)=0

∴a=0, a=-2 Answer

11. Given that $\begin{pmatrix}6 & 8\\2 & 3\end{pmatrix}\begin{pmatrix}w-4\\-1 \quad 3\end{pmatrix}=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

Find the value of w.

Solution

$\begin{pmatrix}6 & 8\\2 & 3\end{pmatrix}\begin{pmatrix}w-4\\-1 \quad 3\end{pmatrix}=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$\begin{Bmatrix}6\times w+8\times -1 & 6\times -4+8\times 3\\2\times w+3\times -1 & 2\times -4+3\times 3\end{Bmatrix}=\begin{Bmatrix}1 & 0\\0 & 1\end{Bmatrix}$

$\begin{Bmatrix}6w-8 & -24+24\\6-3 & -8+9\end{Bmatrix}=\begin{Bmatrix}1 & 0\\0 & 1\end{Bmatrix}$

$\begin{Bmatrix}6w-8 & 0\\2w-3 & 3\end{Bmatrix}=\begin{Bmatrix}1 & 0\\0 & 1\end{Bmatrix}$

Equate corresponding components

Either 6w-8=1

6w=9

$\frac{6w}{6}=\frac{9}{6}$

w=$\frac{3}{2}$

Or 2w-3=0

2w=3

$w=\frac{3}{2}$

$\therefore$ **$w=\frac{3}{2}$ or 1.5 Answer**

12. An aeroplane takes off at an angle of 23^0 to the horizontal ground and flies for 13km. Calculate its height above the ground, leaving your answer correct to 1 decimal place.

Solution

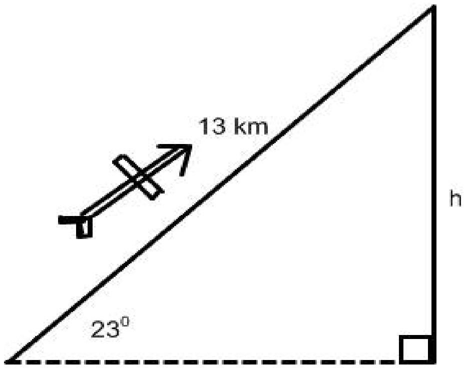


Figure 3

$\frac{h}{13km} = \text{Sin}23^0$

$h = \text{Sin}23^0 \times 13km.$

$=0.39073 \times 13km$

=5.1km Answer

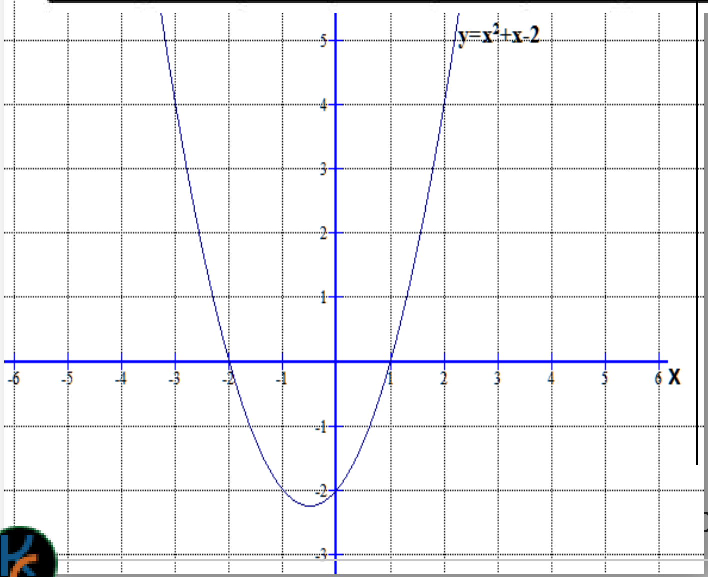
13. **Table 1**, shows the values of **x** and **y** of the equation

$y=x^2+x-2$.

Solution

Table 1

X	-3	-2	-1	0	1	2
y	4	0 ✓	-2	-2	0	4 ✓



Copy and complete the table and sketch the graph of **$y=x^2+x-2$**

Figure 4

14. Solve the simultaneous equations:

$y=x-2$

$xy+1=0$

Solution

$y=x-2$

$xy+1=0$

substitute in the second equation

y by x-2

$x(x-2)+1=0$

$x^2-2x+1=0$

$(x-1)(x-1)=0$

Either x-1=0, x is 1 or x-1=0, or x-1=0, x is 1

$\therefore$When x is 1, y=1-2

16	17	3	13
5	11	10	8
9	7	6	12
4	14	15	17

$y=-1$

$\therefore$ x is 1 twice and y twice is -1 Answer

15. A cylindrical metal bar whose volume is **$594cm^3$** is melted and cast into a sphere. Calculate the radius of the sphere, leaving your answer correct to two decimal places. **(Volume of a sphere=$\frac{4\pi r^3}{3}$, Take π=3.142)**

Solution

Volume of a cylinder= πr^2h

$=\frac{4}{3}\pi r^3=594cm^3.$

$=3.142 \times r^3=594cm^3$

$r^3=\frac{594}{3.142}$

$r=\sqrt[3]{\frac{594}{3.142}}$

=5.739315352

=5.74 to two decimal places

16. Given that n(x)=18, n(y)=24 and n(x∪y)=40. Find n(x∩y).

Solution

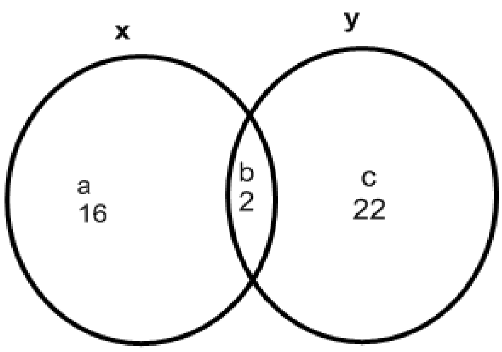


Figure 5

a+b+c=40 (given)

a+b=18 (given)

b+c=24 (given)

a=18-c

b=24-C

Substitution in a+b+c=40

18-c+24-c+c=40

-2c+c=40

-c=-2

C=2

∴n(x∩y)=2 Answer

19. Table 2 shows numbers

If a number is picked at random from the table, calculate the probability that it is prime or less than 9.

Solution

P(P or <9)=P(Prime)+P(<9)-P(P∩<9)

$=\left(\frac{7}{16}\right)+\left(\frac{6}{16}\right)-\left(\frac{7}{16}\times\frac{6}{16}\right)$

$=\frac{13}{16}-\frac{42}{256}$

$=\frac{208-42}{256}$

$=\frac{166}{256}$

$=\frac{\mathbf{83}}{\mathbf{128}}$ **Answer**

20. On the same axes, sketch the graphs of the region described by the inequalities y<3 and y≥-x

Solution

B.L. y<3

y=3

y≥-3

y=-x

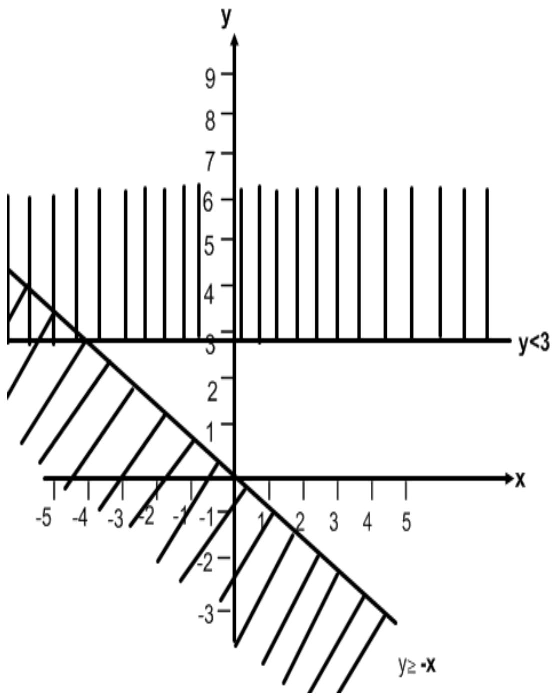


Figure 6

21. Given that q α $\sqrt{p}$ and p=4 when q=3, find the value of p when q=15

Working

$q\alpha\sqrt{p}$

q=k$\sqrt{p}$, k is constant.

3=k$\sqrt{4}$

3=2k

2k=3

∴k=$\frac{3}{2}$

Thus q=$\frac{3}{2}\sqrt{p}$

When q=15

$15\times2=2\times\frac{3}{2}\sqrt{p}$

$30=3\sqrt{p}$

$3\sqrt{p}=30$

$\sqrt{p}=\frac{30}{3}$

$(\sqrt{p})^2=(10)^2$

∴p=100 Answer

21. In a survey conducted at Chitsa village, 20 people responded **"YES," 30** responded, **"NO"** and 10 responded **"DON'T KNOW"** Draw a pie chart to represent the information.

Solution

First calculate angle sectors.

6

"YES"= $\frac{20}{\cancel{60}} \times \cancel{360}^0$

$=120^0$

6

"NO"= $\frac{30}{\cancel{60}} \times \cancel{360}^0$

$=180^0$

6

"DON'T KNOW"= $\frac{10}{\cancel{60}} \times \cancel{360}^0$

$=60^0$

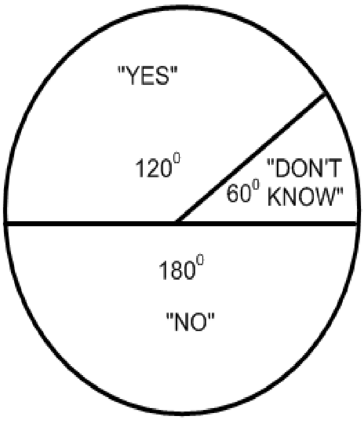


Figure 7

2009 MALAWI SCHOOL CERTIFICATE OF EDUCATION EXAMINATION

MATHEMATIC S SOLUTIONS

1. Factorise completely $2x^2-15x-28$

   Solution

   $2x^2-15x-28$

   Multiply $2x^2$ by 28 (Factors are -7x-8x)

   $2x^2-7x-8x+28$

   x(2x-7)-4(2x-7)

   =(2x-7) (x-4) Answer

2. Given that $f(x)=\frac{3}{3-x}$, find f(-3) in its simplest form.

   Solution

   $f(x)=\frac{3}{3-x}$

   $\therefore f(-3)=\frac{3}{3-(-3)}$

   $=\frac{3}{6}$

   $=\frac{1}{2}$ **Answer**

3. The volume of a pyramid is $60cm^3$ and its base area is $20cm^2$. Calculate the height of the pyramid. (Volume of pyramid=$\frac{1}{3}$base area $\times$ height)

   Solution

   $\frac{1}{3}$base area $\times$ height =Volume of a pyramid

   $\frac{1}{3} \times 20\text{cm}^2 \times \text{height} = 60\text{cm}^3$

   3

   Height=$\frac{\cancel{60\text{cm}^3}}{\cancel{20\text{cm}^2}} \times 3$

   Height = 9 cm Answer

4. Figure 1 shows a circle ABC center O. OCP is a straight line and AP is a tangent to the circle at A. If angle ABC=35^0, calculate the value of angle APO.

   Solution

   Angle ABC=35^0 (given)

   Angle AOP=70^0= (two times angle ABC (angle at centre twice that at circumference)

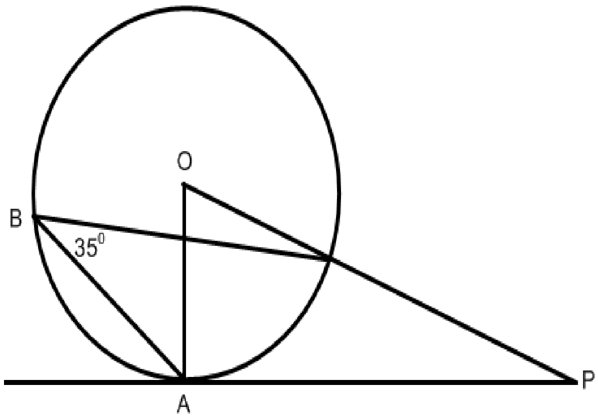


Figure 1

In Triangle APO, angle PAO is 90^0. (Radius OA perpendicular to tangent)

$\therefore$ Angle APO=$180^0-(90^0+70^0)$ (angle sum in a Triangle)

$=180^0-160^0$

$=20^0$ Answer.

5. Without using a calculator or four figure tables, evaluate

$\frac{\text{Tan}60^0}{\text{Tan }30^0}$

Solution

To evaluate $\frac{\text{Tan}60^0}{\text{Tan }30^0}$

$=\frac{\sqrt{3}}{\frac{1}{\sqrt{3}}}$

Or $\sqrt{3}\div\frac{1}{\sqrt{3}}$

$=\frac{\sqrt{3}}{1}\times\frac{\sqrt{3}}{1}$

$=\sqrt{9}$

=3 Answer.

6. In a family six members eat meat, five members eat fish while two members eat both. Calculate the number of members in the family.

Solution

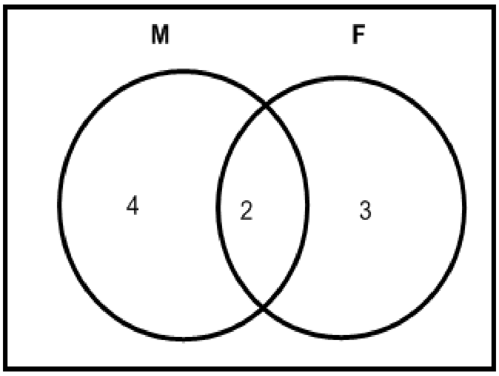


Figure 2

There are 4+2+3

=9 members Answer

7. Solve the equation $3^y=\frac{9^y}{81}$

Solution

$\frac{3^y}{1}=\frac{9^y}{81}$ (cross multiply)

$81\times3^y=9^y$

$81=\frac{9^y}{3}$

$81=\frac{3^{2y}}{3^y}$

$3^4=3^{2y}\div3^y$ (subtract indices)

$3^4=3^{2y-y}$

$3^4=3^y$(equate powers)

∴y=4 Answer.

8. Given that $(x+3)\ (x+1)^2\equiv Ax^3+Bx^2+Cx+D$, find the value of C.

Solution

$(x+3)\ (x+1)^2=(x+3)\ (x+1)\ (x+1)$

$=(x+3)\ (x^2+2x+1)$

$=x^3+2x^2+x+3x^2+6x+3$

$= x^3+2x^2+3x^2+x+6x+3$

$= x^3+5x^2+7x+3$

∴ $x^3+5x^2+7x+3\equiv Ax^3+Bx^2+CX+D$

C is the coefficient of x.

∴ C is 7 Answer.

9. Table 1 shows the distribution of ages of learners in Form 2 class.

Age	14	15	16	17	18	19
Number of Learners	2	10	8	4	9	3

What is the probability of picking at random a learner of 18 years of age?

Solution

Total number of learners

=2+10+8+4+9+3

=36

Total sample space

=36

There are 9 learners of 18 years of age.

∴P (picking learner of 18 years of age)

$=\frac{9}{36}$

$=\frac{1}{4}$ **Answer**

10. Find the gradient of a straight line whose equation is $\frac{y-2x}{4}=\frac{x}{3}$

Solution

$\frac{y+2x}{4}=\frac{x}{3}$ (Cross multiply)

$3(y+2x)= 4x$

$3y+6x= 4x$ (make y subject)

$3y=4x-6x$

$3y=-2x$

$y=-\frac{2}{3}x$

Gradient is coefficient of

$x=-\frac{2}{3}$ Answer

11. Given that $\frac{1}{2}Log_3x=Log_3(6-x)^2$, find the value of x. Find the value of x.

Solution

$\frac{1}{2}Log_3x=Log_3(6-x)^2$(Take antilogs on both sides)

$x^{\frac{1}{2}}=(6-x)^{\frac{1}{2}}$

$\sqrt{x}=\sqrt{6-x}$ (square both sides)

$(\sqrt{x})^2=(\sqrt{6-x})^2$

x=6-x (make x subject)

x+x=6

2x=6

∴x=3 Answer

12. The 9th term of an arithmetic progression y, y+4, y+8,……is 37. Find the value of **y**.

Solution

The 9th term is 37, a=y, d=4, n=9.

Thus a+(n-1)d=37, where a is first term, d is common difference and n is number of terms.

y+(9-1)4=37

y+(8)×4=37

y+32=37

y=37-32

Thus y=5 Answer.

13. Without using a calculator of four-figure tables, simplify

$\frac{11}{5-\sqrt{3}}$, leaving your answer with a rational denominator.

Solution

To simplify $\frac{11}{5-\sqrt{3}}$, multiply both the Numerator and denominator by the conjugate of the denominator, $(5-\sqrt{3})$

$=\frac{11}{(5-\sqrt{3})}\times\frac{(5+\sqrt{3})}{(5+\sqrt{3})}$

$=\frac{11(5+\sqrt{3})}{25-5\sqrt{5}+5\sqrt{5}-3}$

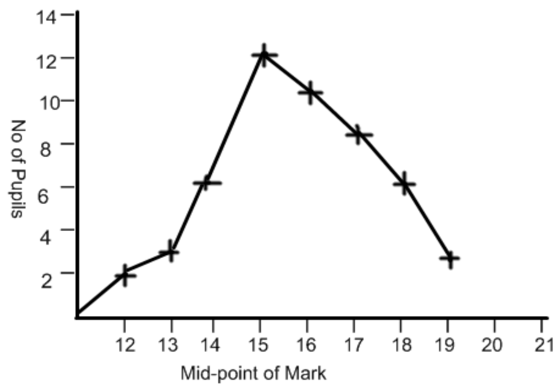
$=\frac{11(5\sqrt{3})}{22}$

$=\frac{5+\sqrt{3}}{2}$ **Answer**

14. A quantity T is proportional to M and the square of V. When V=3 and M=5, T=90. Calculate the value of T when M=2 and V=10.

Solution

Midpoint Mark	12	13	14	15	16	17	18	19
Number of Pupils	2	3	6	12	10	8	6	3



T α MV^2

$T=KMV^2$ (where K is a constant)

90 =k×5×3×3

90 =45K

45k=90

K=2

$∴T=2MV^2$

When m is 2 and V is 10, by substitution,

T=2×2×10×10

=**400 Answer**

15. Express $\frac{1}{x-1}+\frac{3y}{xy-y}$ as a single fraction.

Solution

$\frac{1}{x-1}+\frac{3y}{y(x-1)}$

$=\frac{y(1)+1(3y)}{y(x-1)}$

$=\frac{y+3y}{y(x-1)}$

$=\frac{4y}{y(x-1)}$

$=\frac{4}{(x-1)}$ **Answer**

16. Make r the subject of the formula $P=m\left(\frac{r}{x}\right)^3$

Solution

$P=m\left(\frac{r}{x}\right)^3$ (Divide both sides by m)

$\frac{P}{M}=\left(\frac{r}{x}\right)^3$ (Expand the RHS)

$\frac{P}{M}=\frac{r^3}{x^3}$ (multiply both sides)

$\frac{Px^3}{M}=r^3$

Or

$R^3=\frac{Px^3}{M}$ (Take cube roots on both sides)

$\sqrt[3]{r^3}=\sqrt[3]{\frac{Px^3}{M}}$

$r=x\sqrt[3]{\frac{P}{M}}$

$r = x\sqrt[3]{\frac{P}{M}}$ Answer

17. Table 2 shows midpoints of marks scored by a group of 50 pupils in a test.

    Using a scale of 2cm to represent 1 unit on the horizontal axis and 2cm to represent 2 units on the vertical axis, draw a frequency polygon to represent this information.

18. The areas of two similar triangles ABC and XYZ are in the ratio 1:16. If the height of the smaller triangle is 2 cm, calculate the height of the bigger triangle.

    Solution

    Triangles ABC and XYZ are similar

    $\frac{AB}{XY}=\frac{BC}{YZ}=\frac{CA}{ZX}$

    $\frac{16}{1}=\frac{D}{2cm}$

    Note: Area of similar triangles is the ratio of squares of corresponding sides

    $\frac{16}{1}=\frac{(h)^2}{(2)^2}$

    $\frac{16}{1}=\frac{d^2}{4}$ (cross multiply)

    $16\times4=h^2\times1$

    $64=h^2$

    Or $h^2=64$ (Take square roots from both sides)

    $h=8$

    ∴ The height of the bigger triangle is 8 cm Answer.

19. On the same axes, sketch the graphs of the region described by the inequalities 2<x≤5 and 0≤4. Shade the unwanted region.

    Solution

    For 2<x≤5, Boundary lines x=2 and x=5.

    For 0≤y<4, Boundary lines are y=0 and y=4.

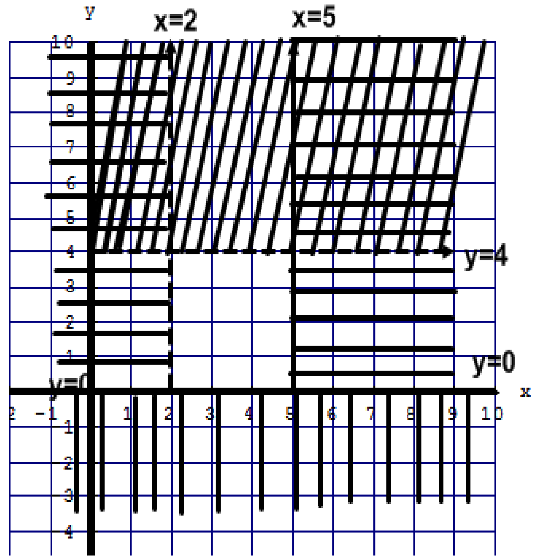


Figure 3

20. A and B are two matrices. If $A=\begin{pmatrix}0 & 1\\3 & 2\end{pmatrix}$, find B given that $A^2=A+B$.

    Working

    $A+B=A^2$

    $B=A^2-A$

    $=\begin{pmatrix}0 & 1\\3 & 2\end{pmatrix}\times\begin{pmatrix}0 & 1\\3 & 2\end{pmatrix}-\begin{pmatrix}0 & 1\\3 & 2\end{pmatrix}$

$=\begin{pmatrix} 0\times0+1\times3 & 0\times1+1\times2 \\ 3\times0+2\times3 & 3\times1\times2\times2 \end{pmatrix}-\begin{pmatrix} 0 & 1 \\ 3 & 2 \end{pmatrix}$

$=\begin{pmatrix} 0+3 & 0+2 \\ 0+6 & 3+4 \end{pmatrix}-\begin{pmatrix} 0 & 1 \\ 3 & 2 \end{pmatrix}$

$=\begin{pmatrix} 3 & 2 \\ 6 & 7 \end{pmatrix}-\begin{pmatrix} 0 & 1 \\ 3 & 2 \end{pmatrix}$ (Subtract corresponding terms)

$=\begin{pmatrix} \mathbf{3} & \mathbf{1} \\ \mathbf{3} & \mathbf{5} \end{pmatrix}$ **Answer**

21. Figure 4 shows a graph of a quadratic equation $y=ax^2+bx+c$

 Find the equation for the graph in the form

 Figure 4

 $y=ax^2+bx+C$

 Solution

 The curve $y=ax^2+bx+c$

 Cross x axis at two points when y=0, where x= -1 and x=2.

 At these two points, when y=0, the roots of the equation

 $0=ax^2+bx+c$, are -2 and +1.

 Thus (x-2)(x+1)=y

 Expand the LHS

 $x^2+x-2x-2=y$

 $x^2-x-2=y$

 ∴ Equation is $y=x^2-x-2$ Answer

2010 MALAWI SCHOOL CERTIFICATE OF EDUCATION EXAMINATION

MATHEMATIC S SOLUTIONS

1. Factorise completely $2x^2+3xy-35y^2$

 Solution

 $2x^2+3xy-35y^2$

 Multiply the two ($2x^2$ by $-35y^2$)= $-70x^2y^2$

 $=2x^2+10xy-7xy-35y^2$

 =2x(x+5y)-7y(x+5y)

 =(x+5y)(2x-7y) Answer

2. Given that X={a,c,e}, Y={b,c,d,e} and Z={c,d,e,f}. Find (X∪Y)∩Z.

 Solution

 (X∪Y)={a,b,c,d,e}

 Z={c,d,e,f}

 ∴(X∪Y)∩Z

 ={c,d,e) Answer

3. Figure 1 shows a circle WXYZ center O. Angle WOY=112^0 and angle XWY=36^0.

 Calculate the size of angle WZX.

 Solution

 Angle WZY=$\frac{1}{2}$ Angle WOY (angle centre twice at circumference)

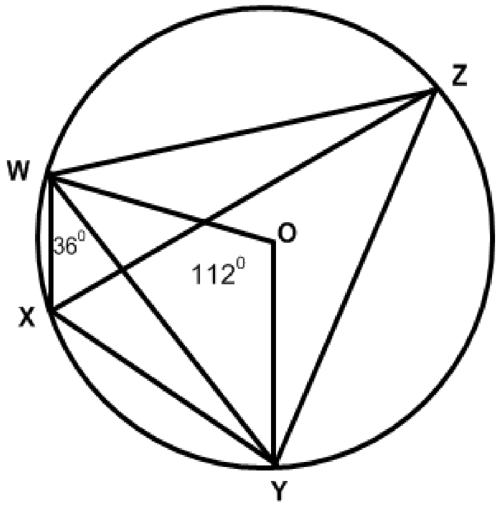


 Figure 1

 $=\frac{1}{2}\times 112^0$

 $=56^0$

 Angle WXY=180^0-56^0 (WXYZ is cyclic quadrilateral)

 $=124^0$

 In Triangle WXY, angle WYX is 180^0-(124^0+36^0) (angle sum in a triangle)

 $=180^0-160^0$

 $=20^0$

 Angle WZX=Angle WYX==20^0(angles in same segment)

 ∴Angle WZX==20^0 Answer

4. The function f(y)=3y+2. Given that {5) is the range, find the domain.

Solution

f(y)=3y+2

5=3y+2

Or 3y+2=5

3y=5-2

3y=3

y=1

∴ Domain is {1} Answer.

5. Without using a calculator or four-figure tables, simplify $\frac{\sqrt{54}+3\sqrt{3}}{\sqrt{3}}$ in its simplest form.

Solution

$\frac{\sqrt{54}+3\sqrt{3}}{\sqrt{3}}$

$=\frac{\sqrt{9\times3\times2}+3\sqrt{3}}{\sqrt{3}}$

$=\frac{3\sqrt{3\times2}+3\sqrt{3}}{\sqrt{3}}$

$=\frac{3\sqrt{3\times2}+\sqrt{3\times1}}{\sqrt{3}}$

$=\frac{3\times\sqrt{3}(\sqrt{2}+\sqrt{1})}{\sqrt{3}}$

$=3\sqrt{2}+1$ Answer

OR

$\frac{\sqrt{54}+3\sqrt{3}}{\sqrt{3}}$ (Divide each term in numerator by$\sqrt{3}$)

$=\sqrt{18}+3$

$=\sqrt{9\times2}+3$

$=3\sqrt{2}+3$

$=3(\sqrt{2}+1)$ Answer

6. A geometric progression has 6 terms. If the first term is 96, calculate the common ratio.

Solution

The n^{th} term of a **GP=ar^{n-1}** where a is first term, **r** is common ratio, n is the number of terms.

First Term, **ar^{n-1}=3**

That is **ar^{n-1}=3**

a=3

Last Term, i.e. 6^{th} term= 96

ar^{6-1}=96

ar^5 =96

$3\times r^5$=96

$r^5=\frac{96}{3}$

$r^5=32$

$r^5=2\times2\times2\times2\times2$

$r^5=2^5$(Equate bases)

∴ The common ratio is 2 Answer.

7. Given that matrix p=$\begin{pmatrix}7 & 5\\ 2 & 4\end{pmatrix}$ and Q=$\begin{pmatrix}3 & 10\\ 5 & 1\end{pmatrix}$, find PQ

Solution

$PQ=\begin{pmatrix}7 & 5\\ 2 & 4\end{pmatrix}\times\begin{pmatrix}3 & 10\\ 5 & 1\end{pmatrix}$

$=\begin{pmatrix}7\times3+5\times5 & 7\times10+5\times1\\ 2\times3+4\times5 & 2\times10+4\times1\end{pmatrix}$

$=\begin{pmatrix}21+25 & 70+5\\ 6+\ 20 & 20+\ 4\end{pmatrix}$

$=\begin{pmatrix}46 & 75\\ 26 & 24\end{pmatrix}$Answer

8. Given that Log_5 x+Log_5 y=3Log_5q, show that

$X=\frac{q^3}{y}$

Solution

Log_5 x+Log_5 y=3Log_5q

Log_5 (xy)= 3Log_5q (Take antilogs on both sides)

$(xy)=q^3$

(divide both sides by y making x subject of the formula)

$W=\frac{q^3}{y}$ as required Answer

9. Figure 2 shows a straight line passing through a point P(3,4)

Given that tan Ø $=\frac{2}{3}$, find the equation of the line in the form y=mx+c

Solution

Equation of the line $y-y_1=m(x-x_1)$, where m is $\frac{2}{3}$, y_1 is 4 and x_1 is 3

$y-4=\frac{2}{3}(x-3)$

$y-4=\frac{2}{3}x-3$

$y=\frac{2}{3}x-3+4$

$y=\frac{2}{3}x+1$

$y=\frac{2}{3}x+1$ Answer

10. Show that K+3 is a factor of $k^3+3k^2-4k-12$

Solution

Let k+3=0

Substitute k by -3 is in $k^3+3k^2-4k-12$

$=(-3)^3+3(-3)^2-(4\times-3)-12$

=-27+27+12-12

=27-27+12-12

=0+0

=0

∴k+3 is a factor Answer.

11. The results of a test marked out of 25 written by 20 learners were as follows:

1	7	13	12	14
12	18	17	19	17
17	19	22	23	24
22	22	24	23	22

Using class intervals of 1-5, 6-10, 11-15,..., construct a frequency table for the results.

Solution

Table 1

Class Interval	Tally	Frequency
1-5	I	1
6-10	I	1
11-15	IIII	4
16-20	卌I	6
20-25	卌III	8

12. Figure 2 shows two similar triangles TQU and RSU, TU=3 cm, UR=6cm and RS is parallel to QT.

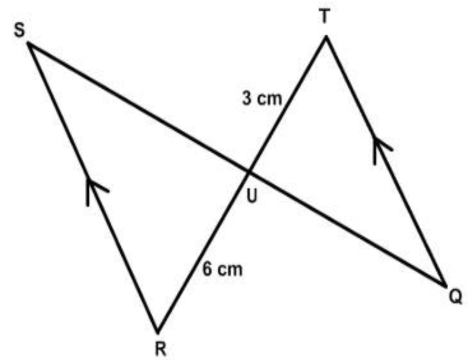


Figure 2

Calculate the ration of the area of triangle TQU to the area of triangle RSU, leaving your answer in its simplest form.

Solution

Tringles TQU and RSU are similar

$\frac{TQ}{RS}=\frac{QU}{SU}=\frac{UT}{UR}$

$\frac{TQ}{RS}=\frac{QU}{SU}=\frac{3cm}{6cm}$

The areas of two similar triangles is the squares of ratio of corresponding sides.

$\frac{(3)^2cm^2}{(6)^2cm^2}$

$=\frac{3\times3}{6\times6}$

$=\frac{9}{36}$

$=\frac{1}{4}$

=1:4 Answer

13. Make a subject of the formula:

$X=\frac{\sqrt{a}}{y}+b$

Solution

$X=\frac{\sqrt{a}}{y}+b$ (Multiply each fraction by y as common denominator)

$\frac{x\times y}{1}=\frac{\sqrt{a}}{y}\times y+\frac{b}{1}\times y$

$xy=\sqrt{a}+by$

$xy-by=\sqrt{a}$

$(y(x-b))^2=(\sqrt{a})^2$ (square both sides)

$(y(x-b))^2=a$

∴$a=(y(x-b))^2$ Answer.

14. Simplify $\frac{d-1}{3}-\frac{2d+1}{7}$

Solution

$\frac{d-1}{3}-\frac{2d+1}{7}$ (Common denominator is 21)

$=\frac{7(d-1)-3(2d+1)}{21}$

$=\frac{7d-7-6d-3}{21}$

$=\frac{7d-6d-7-3}{21}$

$=\frac{\mathbf{d-10}}{\mathbf{21}}$ **Answer**

15. Figure 5 is a graph of a quadratic function.

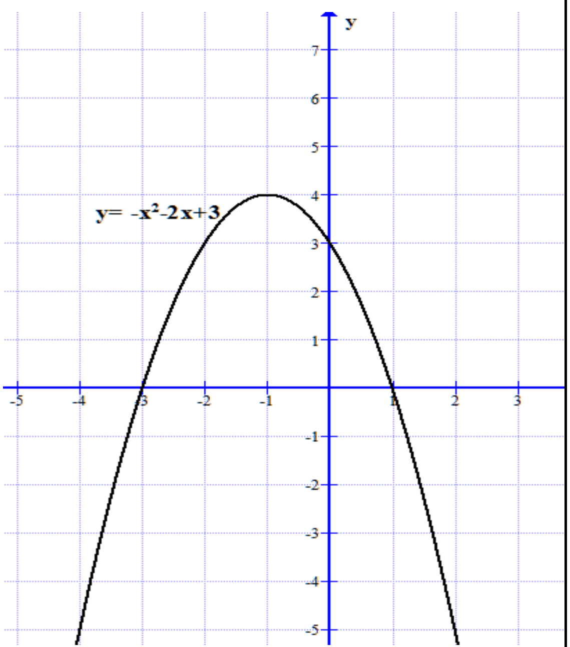


Figure 3

Formulate an equation of the graph in the form y=ax+bx+c

Solution

Let the roots be +3 and -1

Thus (x+3) (x-1)=0

x^2-x+3x-3

x^2+2x-3

But the curve is cape shaped parabola

∴Multiply (x^2+2x-3) ×-1

= -x^2-2x+3

∴y=-x^2-2x+3Answer.

16. Figure 6 is a tree diagram which shows the probability of picking two balls one at a time without replacement from a bag containing 6 red and 4 white balls.

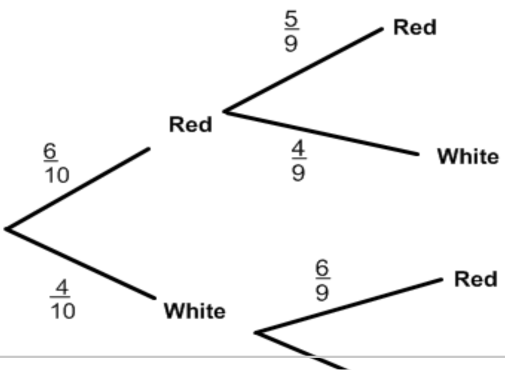


Figure 4

Use the tree diagram to calculate the probability of picking two balls of different colours, leaving your answer in its simplest form.

Solution

P(two balls of different colours)

$=\left(\frac{6}{10}\times\frac{4}{9}\right)+\left(\frac{4}{10}\times\frac{6}{9}\right)$

$=\frac{24}{90}+\frac{24}{90}$

$=\frac{48}{90}$

$=\frac{16}{30}$

$=\frac{\mathbf{8}}{\mathbf{15}}$ **Answer**

17. Figure 5, shows a rectangular based right pyramid FGHIJ. GH=10cm and HI=7 cm. If the height of the pyramid is 13cm, calculate the angle between FHI and the base.

Solution

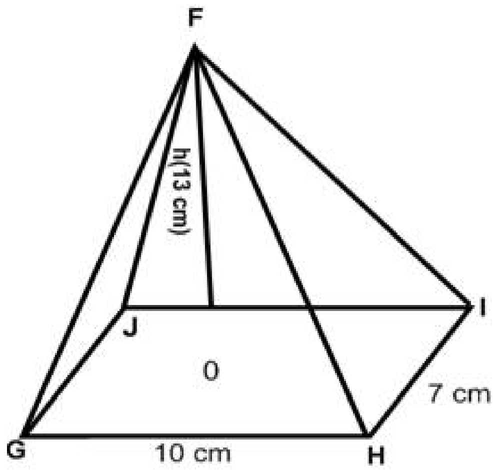


Figure 5

Construction: Drop a perpendicular from angle HFI to meet HI at K. Join OK.

In Triangle FOK, OK= 5 cm angle between FHI and base is angle FKO

$=\frac{13cm}{5cm}$=Tan ∅

=2.6

Tan $^{-1}$ 2.6= 69^{0} to the nearest degree

Answer

18. On the same axes, sketch the graphs of the region described by the following inequalities.

x≥0

y=0

y≤3x+2

y+4x<8

Shade the unwanted region

Solution

Boundary lines for

x≥0, x=0

y=0, y=0

y≤3x+2, y=3x+2

y+4x<8, y+4x=8

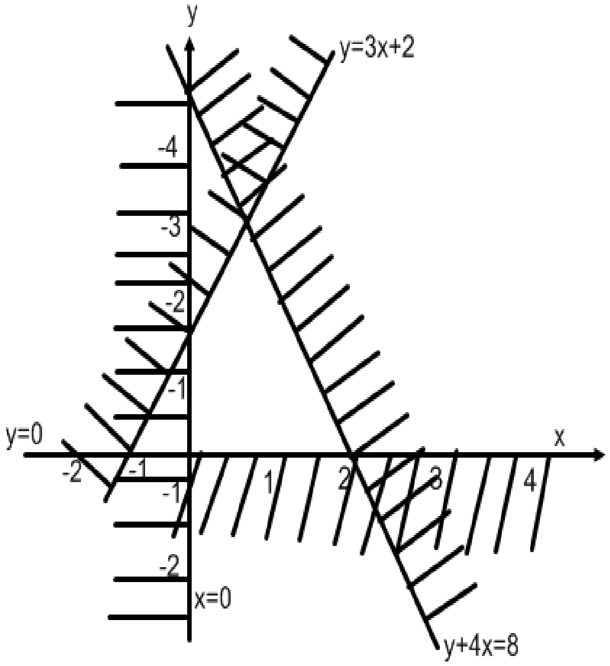


Figure 6

19. Given that Vα rd and V=54 when r=2, and d=3. Find V when r=$\frac{1}{2}$ and d=6.

Solution

Vα rd

V=K rd where K is constant.

54=k×2×3

$\frac{54}{6}=\frac{6K}{6}$

K=9

∴V=9rd

When r is $\frac{1}{2}$ and d=6

V=$\frac{9\times1\times6}{2}$

=27 Answer.

20. A circle KLM has centre O. The diameter KM=15cm and chord KL=12cm. E is a point on LM. If OE is perpendicular to the chord LM, calculate the length of EM.

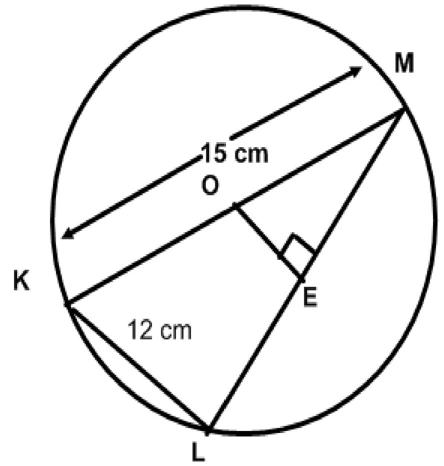


Figure 7

Solution

In triangles MOE and MKL

Angle OME=angle KML (common angle)

Angle OEM=angle KLM=90^{0}.

∴Third angles are equal.

Thus Δs MOE and MKL are ≡ (are equiangular)

$\therefore\frac{MO}{MK}=\frac{OE}{KL}=\frac{ME}{KL}(\frac{7.5}{15}=\frac{OE}{12})$

$\sqrt{MO^2-OE^2}$=$\frac{OF}{EM}$ (Pythagoras theorem)

=$\frac{12\times7.5}{15}$=6 cm

= $\frac{6}{\sqrt{20.25}}$

=4.5 cm Answer

21. Figure 8 shows an isosceles triangle XYZ with sides XY=XZ=9cm and YZ=12cm. XD is a perpendicular bisector of YZ.

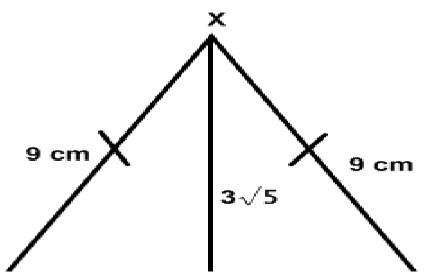


Figure 8

Calculate angle YXZ giving your answer correct to the nearest degree.

Solution

YD=DZ=6cm (XD perpendicular to YZ)

Each Triangle XDY and XDZ is a right angled type. (XD is perpendicular to YZ)

$XD^2=XY^2-DY^2$

$=9^2-6^2$

=81-36

=45

$\therefore XD=\sqrt{45}$

$=\sqrt{9\times5}$

$=3\sqrt{5}$

Tan Angle YXD$=\frac{6}{3\sqrt{5}}=\frac{6}{6.708}$

=0.89445

$=41.8^0$

Angle YXZ=41.8 ×2

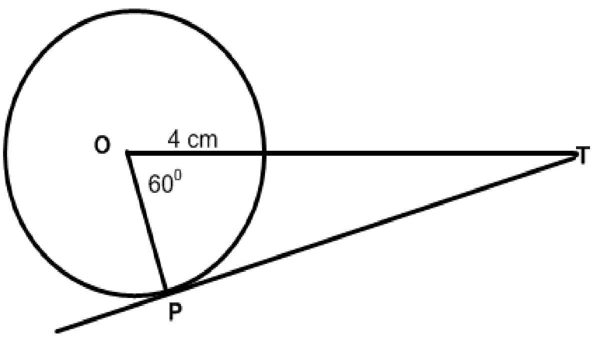
$=83.6^0$ (DX bisects angle YXZ)

∴ Angle YXZ=twice angle YXD.

22. Using a ruler and compass only, construction in the same diagram.
    - (i) A circle centre O of radius 4 cm.
    - (ii) A tangent TP to the circle at any point P such that angle POT$=60^0$.
    - (iii) Measure and state the length PT.

    Working

    Sketch.



2011 MALAWI SCHOOL CERTIFICATE OF EDUCATION EXAMINATION

MATHEMATIC S SOLUTIONS

1. Factorise completely $3-5x-2x^2$.

   Solution

   $3-5x-2x^2$

   (Multiply factors -6x by x)

   $=-6x^2$.

   $=3-5x-2x^2$

   =3(1-2x)+x (1-2x)

   =(1-2x) (3+x) Answer

2. Given that $y(x) = \frac{2x^3}{3}+1$, find g(-1) in its simplified form.

   Solution

   $g(x)=\frac{2x^3}{3}+1$

   $g(-1)=\frac{2x^3}{3}+1$

   $g(-1)=\frac{2\times(1)^3}{3}+1$

   $=\frac{2\times(-1\times-1\times-1)}{3}+1$

   $=-\frac{2}{3}+1$

   $=\frac{-2+3}{3}$

   $=\frac{1}{3}$ **Answer**

3. Without using a calculator or four-figure tables, simplify $\frac{1}{\sqrt{2}}-\frac{\sqrt{2}}{3}$, leaving your answer with a rational denominator.

   Solution

   $\frac{1}{\sqrt{2}}-\frac{\sqrt{2}}{3}$ (LCM of $\sqrt{2}$ and 3 is $3\sqrt{2}$)

   $=\frac{3-\sqrt{2}}{3\sqrt{2}}\times\frac{3\sqrt{2}}{3\sqrt{2}}$

   $=\frac{9\sqrt{2}-3\times\sqrt{2}\times\sqrt{2}}{3\times3\times\sqrt{2}\times\sqrt{2}}$

   $=\frac{9\sqrt{2}-3\times2}{9\times2}$

   $=\frac{9\sqrt{2}-6}{9\times2}$

   $=\frac{\cancel{3}(3\sqrt{2}-2)}{\cancel{18}\ 6}$

   $=\frac{3\sqrt{2}-2}{6}$ **Answer**

4. Given that $Log_x 6\frac{1}{4}$=2, solve for x.

Solution

$Log_x 6\frac{1}{4}$ =2

(change to exponential equation).

$6\frac{1}{4} = x^2$

$\frac{25}{4} = \frac{x^2}{1}$

$4x^2=25$

$x^2=\frac{25}{4}$

$x=\sqrt{\frac{25}{4}}$

$x=\frac{5}{2}$ Answer

5. The ratio of area of two circle is 4:9. Given that the radius of the bigger circle is 18 cm, find the radius of the smaller circle.

Solution

Ratio of area=4:9

Radius of bigger circle is 18 cm

i.e. $\frac{4}{9} = \frac{r}{18}$

(Area factor)$\left(\frac{2}{3}\right)^2 = \frac{r}{18}$(Scale factor)

$\frac{2}{3} = \frac{r}{18}$

3r=2×18

r=2×18

6

$r=2\times\frac{18}{3}$

=12 cm Answer.

6. Make y subject of the formula $p=\sqrt{\frac{y-a}{y+1}}$

Solution

$P=\sqrt{\frac{y-a}{y+1}}$ (Square both sides)

$(p)^2=\left(\sqrt{\frac{y-a}{y+1}}\right)^2$

$p^2=\frac{y-a}{y+1}$ (Cross multiply)

$p^2(y+1)=y-a$

$p^2y+q^2=y-a$

$p^2y-y=-p^2-a$

$y(p^2-1)=-p^2-a$

$y=\frac{-p^2-a}{p^2-1}$

$\therefore y=\frac{-p^2-a}{(p+1)(p-1)}$ Answer

7. The mean of (x-1) (x+2) and (x+5) is 2x. Find the value of x.

Solution

$\frac{(x-1)+(x+2)+(x+5)}{3} = 2x$ (Multiply both sides by 3)

x-1+x+2+x+5= 6x

x+x+x+5+2-1= 6x

3x+6=6x

3x-6x=-6

-3x=-6

x=2 Answer

8. Simplify $\frac{M}{M+2} - \frac{6}{M^2+M-2}$

Solution

$\frac{M}{M+2} - \frac{6}{(M+2)(M-1)}$

$= \frac{M}{M+2} = \frac{6}{(M+2)(M-1)}$

$= \frac{M(M-1)-6}{(M+2)(M-1)}$

$=\frac{M^2-M-6}{(M+2)(M-1)}$

$=\frac{(M-3)\cancel{(M+2)}}{\cancel{(M+2)}(M-1)}$

$=\frac{M-3}{M-1}$ **Answer**

9. Given that (x+1) and (x-3) are two factors of the polynomial ax^3+bx-6, calculate the values of a and b.

Solution

Let x+1=0

x=-1

$p(-1)=a(-1)^3 + b(-1) - 6$

0= -a-b-6

a+b= -6

Also let x-3=0

x=3

$p(3)=a(a)^3 + b(3) - 6$

0=27a+3b-6

-27a-3b=-6

a+b=-6

(-27a-3b=-6)÷3

a+b=-6

-9a-b=-2 (add the two equations)

-8a=-8

A=1

a+b=-6

1+b=-6

b=-6-1

=-7

∴ a is 1 and b is -7 Answer

10. Figure 1 shows a venn diagram.
    In the venn diagram
    ξ= {girls in form three}
    N={girls that play netball}
    V={girls that play volley ball}
    Given that there are 21 girls in class, find how many girls play both netball and volleyball.

Solution

ξ =2x+2x+1+x+10.

21=2x+2x+x+1+10

21=5x+11

21-11=5x

10=5x

2=x

∴x=2

∴ No that plays both netball and volleyball

=2x+1

=(2×2)+1

=4+1

=5 Answer.

11. T and R are two matrices, Given that $T=\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ and $R=\begin{pmatrix} 0 & 3 \\ -1 & 3 \end{pmatrix}$, Find 3 R-$T^2$

Solution

$3\begin{pmatrix} 0 & 3 \\ -1 & 1 \end{pmatrix}-\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$

$=\begin{pmatrix} 0 & 9 \\ -3 & 3 \end{pmatrix}-$

$\begin{pmatrix} 2\times 2+1\times -1 & 2\times 1+1\times 3 \\ -1\times 2+3\times -1 & -1\times 1+3\times 3 \end{pmatrix}$

$=\begin{pmatrix} 0 & 9 \\ -3 & 3 \end{pmatrix}-\begin{pmatrix} 4-1 & 2+3 \\ -2-3 & -1+9 \end{pmatrix}$

$=\begin{pmatrix} 0 & 9 \\ -3 & 3 \end{pmatrix}-\begin{pmatrix} 3 & 5 \\ -5 & 8 \end{pmatrix}$

$=\begin{pmatrix} 0-3 \\ -3+5 \end{pmatrix}-\begin{pmatrix} 9-5 \\ 3-8 \end{pmatrix}$

$=\begin{pmatrix} -3 & 4 \\ 2 & -5 \end{pmatrix}$ **Answer**

12. A straight line which passes through (3t, 7) and (t, -5) has gradient 3. Find the equation of the line.

Solution

Gradient=$\frac{y_2-y_1}{x_2-x_1}$

$=\frac{-5-7}{t-3t}$

$=\frac{-12}{-2t}$

$=\frac{6}{t}$

Now $\frac{6}{t}=3$

∴6=3t

t=2

∴points are (3t, 7) and (2,-5)

=(6,7) (2,-5)

∴ Equation of the line is $y-y_1=m(x-x_1)$

Where m=3, $y_1=7$ and $x_1=6$

y-7=3(x-6)

y-7=3x-18

y=3x-18+7

y=3x-11 Answer

Or

y-(-5)=3(x-2)

y+5=3x-6

y=3x-6-5

y=3x-11 Answer

13. P varies directly as r and inversely as the square root of q. Given that p-4 when q=9 and r=1, calculate q when r=2 and p=6.

Solution

$P\alpha\frac{r}{q^2}$

$P=\frac{Kr}{q^2}$ (K constant)

$4=\frac{K\times 1}{9\times 9}$

$4=\frac{K}{81}$

∴K=324

Thus $p=\frac{324r}{q^2}$

When r=2 and p=6, calculating q,

$6=\frac{324\times 2}{q^2}$

$6q^2=324\times 2$

$q^2=\frac{324\times 2}{6}$

$q^2=56\times2$
$q^2=112$
q=$\sqrt{112}$

q=$\sqrt{16\times7}$

q=4$\sqrt{7}$ Answer

14. A tangent DE touches a circle ABCD at D.AC is parallel to DE and angle ADC=30^0 Calculate the value of x

Solution

Angle ACD=angle ABD=X (angles in the same segment)

Angle ACD=angle CDE=x

(Alt angles AC//DE)

Also angle ADF=angle ACD=x (Alt segment)

∴X+130^0+x=180^0(adjacent angles on straight line)

2x=180^0-130^0

2x=50^0

X=50^0

15. The 3^{rd} and 9^{th} terms of an arithmetic progression are 29 and 8 respectively. Calculate the $20^{th.}$ term of the progression.

Solution

n^{th} term of an AP=

29=a+d (3-1)

29=a+2d…………………1

8=a+d (9-1)

8=a+8d…………………2

29=a+2d

-8=a+8d

21=-6d

-6d=21

d=$\frac{21}{-6}$

-$\frac{7}{2}$=$-3\frac{1}{2}$

29=a-$\frac{7}{2}$×2

29+$\frac{7}{2}$=a

36=a

n^{th}=36-$\frac{7}{2}(n-1)$

∴20^{th} term=36 -$\frac{7}{2}(20-1)$

=36-($\frac{7}{2}$×19)

=36-66.5

=-30.5 Answer

16. In figure 2, ABC is a triangle such that angle ABC=460, AB=7 cm and BC=9 cm.

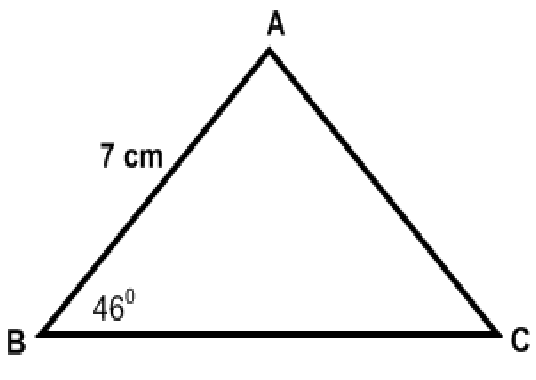


Figure 2

Calculate the length of AC to one decimal place.

Solution

By notation

AC=b

AB=c

BC=a

By Cosine Rule

$\therefore b^2=a^2+c^2-2ac \text{ Cos } B$

$B^2=9^2+7^2-2\times9\times7 \text{ Cos } 46^0$.

$=81+49-126 \text{ Cos } 46^0$

=130-126×0.6946

=130-87.5196

=42.4804

$=42.5^0$ Answer.

17. Sketch the region represented by the following inequalities by shading the unwanted region

x≥-1

y≥4

y≤2x+4

y+2x≤6

Solution

Boundary lines for

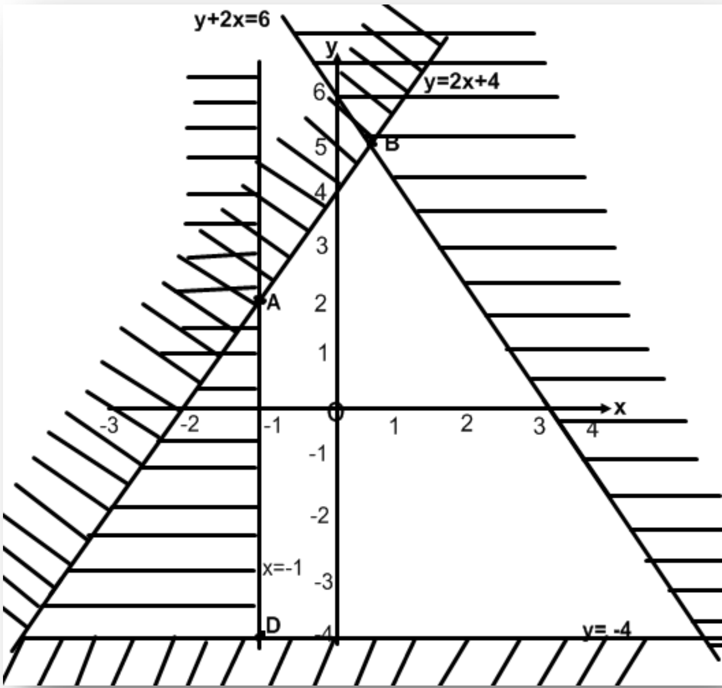
x≥-1, is x=-1

y≥4, is y= -4

y≤2x+4 is y=2x+4

y and x intercepts $\begin{pmatrix} x & y \\ 0, & 4 \\ -2, & 0 \end{pmatrix}$

y+2x≤6 is y+2x=6 y and x intercepts $\begin{pmatrix} x & y \\ 0, & 6 \\ 3, & 0 \end{pmatrix}$



Feasible region is a Quadrilateral ABCD.

19. A pond is 12 m in diameter, has a shape of a hemisphere and is full of water. The pond is emptied and all the water poured into a cylindrical tank of radius 5 cm. Assuming there is no loss of water, calculate the height of water in the tank. (Volume of sphere=$\frac{4}{3}\pi r^3$)

Solution

$\frac{4}{3}\pi r^3$ **= (Volume of sphere)**

$\frac{4}{3}\times\frac{\pi r^3}{2}$ **= Volume of hemisphere**

$\frac{4}{3}\times\frac{3.14\times r^3}{2}$ **= Volume of hemisphere**

$\frac{4\times3.14\times\overset{2}{\cancel{6}}\times6\times6}{\underset{1}{\cancel{3}}\times\underset{1}{\cancel{2}}}$

$=2\times3.14\times2\times6\times6 \text{ cm}^3$

Volume of a cylinder is

Πr^2h.

$3.14 \times 5 \times 5 \times h = 2 \times 3.14 \times 720000$

$h = \frac{2 \times \cancel{3.14} \times 720000}{\cancel{3.14} \times 5 \times 5}$ cm

h=144÷25cm

```
      5.76
h=25√144
      125
      190
      175
      150
      150
```

∴height is 5.76metres

20. Given that $\underline{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} 9 \\ 4 \end{pmatrix}$, calculate $|\underline{a} + \underline{b}|$

Working

$= \underline{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} 9 \\ 4 \end{pmatrix}$

$= |\underline{a} + \underline{b}|$

$= \sqrt{\begin{vmatrix} 3+9 \\ 5+4 \end{vmatrix}}$

$= \sqrt{\begin{vmatrix} 12 \\ 9 \end{vmatrix}}$

$= \sqrt{(12)^2 + (9)^2}$

$= \sqrt{144 + 81}$

$= \sqrt{225}$

=15 Answer